ACTA SCIENTIARUM POLONORUM
Acta Sci. Pol., Formatio Circumiectus 10 (2) 2011, 47–56

ON USING THE SNYDER AND CLARK UNIT HYDROGRAPH FOR CALCULATIONS OF FLOOD WAVES IN A HIGHLAND CATCHMENT (THE GRABINKA RIVER EXAMPLE)

Andrzej Wałęga
Uniwersytet Rolniczy w Krakowie

Magdalena Grzebinoga, Bartłomiej Paluszkiewicz
MGGP S.A. Oddział w Krakowie

Abstract. Using the highland catchment of the Grabinka river located in the Wisłoka drainage basin as an example we assessed the capability of Snyder's and Clark's synthetic unit hydrograph (SUH) to simulate flood wave. Calibration for model parameters was based on a rainfall episode recorded in June 2006. We adopted the minimum of the objective function as an optimisation criterion. The quality of the models was evaluated using the efficiency coefficient *E*. Analysis showed that both Snyder's and Clark's SUH describe properly the observed wave, with the first model yielding somewhat better results. For both SUH the times to culmination were the same as for the observed wave, whereas the calculated culmination discharge differed from the observed one: for Snyder's SUH it was 0.11% higher, and for Clark's SUH 1.9% lower than the observed discharge.

Key words: synthetic unit hydrograph, objective function, optimisation

INTRODUCTION

Since the dawn of civilisation, destructive floods have threatened settlements located in river valleys and plains. Despite developments in technology and extensive investments in flood control works, flood occurrences and accompanying hardships and material damages are not decreasing. The global flood losses have grown worldwide to the level of billions of US dollars per year [Nandalal 2009, after Kundzewicz 2001].

Forecasting floods based on mathematical modelling allows experts to convert information on the past-to-present rainfall into a river flow forecast (discharge, stage, and

Corresponding author – Adres do korespondencji: dr inż. Andrzej Wałęga, Katedra Inżynierii Sanitarnej i Gospodarki Wodnej, Uniwersytet Rolniczy w Krakowie, al. Mickiewicza 24/28, 30-059 Kraków; e-mail: a.walega@ur.krakow.pl

inundated area) for a future time horizon. It helps to reduce flood damage by permitting the public to act before the flood level increases to a critical level. Prognosis of the size of maximum discharge and discharge hydrographs can be created using a rainfall-runoff model. In hydrological works we often use conceptual models (eg. Nash or Wackermann model) [Soczyńska 1997]. The values of model parameters are estimated using recorded episodes of rainfall-runoff. In many parts of the world, rainfall and runoff data are seldom adequate to determine a unit hydrograph of a basin or watershed. In the absence of rainfall-runoff data, unit hydrographs can be derived by synthetic means [Limantara 2009]. A synthetic unit hydrograph is a unit hydrograph derived using an established formula, without a need for analysing the rainfall-runoff data [Ponce 1989]. This includes Snyder's method, Soil Conservation Service (SCS) method, Gray's method and Clark's Instantaneous Unit Hydrograph method. The peak discharges of stream flow from rainfall can be obtained from the design storm hydrographs developed from unit hydrographs generated by established methods [Salami et al. 2009]. Parameters in the methods mentioned above are estimated on the basis of regional regression equations [Straub et al. 2000, Belete 2009]. These equations are unfortunately often prepared for drainage basins and climates substantially different from those found in Poland. Because of that we find it necessary to evaluate the capability of using synthetic unit hydrographs for basins in Poland and to verify correlations for determining SUH parameters.

The aim of this work is to evaluate the capability of using Snyder's and Clark's synthetic unit hydrographs for simulation of rainfall discharges in the highland of the Vistula's Carpathian basin.

MATERIAL AND METHODS

Observed Unit Hydrograph

The basis for calculations was a rainfall-runoff episode recorded in June 2006 in the Głowaczowa cross-section, closing off the Grabinka river basin. Rainfall data was acquired in Tarnow. Rainfalls and discharges data has been made available by IMGW in Krakow [Dane hydrologiczne… 2009]. It was necessary to interpolate rainfall over basin area using rainfall reduction curves in time and area function [Ponce 1989] because of the discreteness of rainfall data. Rainfalls and discharges were analysed with a time step $\Delta t = 12$ h. Base flow was separated from the hydrograph with recession method [Soczyńska 1997]. Effective rainfall which produced flood was analysed with SCS method. In our research the value of *CN* parameter was established by optimisation, using the recorded episode of rainfall-runoff [Soczyńska et al. 2003].

Synthetic Unit Hydrograph (SUH)

Snyder unit hydrograph

In the year1938, Snyder introduced a concept of the synthetic unit hydrograph. An analysis of a large number of hydrographs from catchments in the Appalachian region led to the following formula for lag [Ponce 1989]:

$$T_{\text{lag}} = C_t (LL_c)^{0.2} \tag{1}$$

where:

- T_{lag} – catchment lag in hours,
- C_t – coefficient accounting for catchment gradient and associated catchment storage,
- L – length along the mainstream from outlet to divide (km),
- L_c – length along the mainstream from outlet to point closest to catchment centroid (km).

Snyder's formula for peak flow is as follows [Ponce 1989]:

$$Q_p = \frac{2.78 \cdot C_p \cdot A}{T_{\text{lag}}} \tag{2}$$

where:

- Q_p – unit hydrograph peak flow corresponding to 1 cm of effective rainfall ($m^3 \cdot s^{-1}$),
- A – catchment area (km^2),
- C_p – empirical coefficient relating triangular time base to lag.

Clark unit hydrograph

Clark [1945] developed a method for generating unit hydrographs for a watershed based on routing a time-area relationship through a linear reservoir. Excess rainfall covering a watershed to some unit depth is released instantly and allowed to traverse the watershed, and the time-area relation represents the translation hydrograph. The time--area relationships are usually inferred from a topographic map. The linear reservoir is added to reflect storage effects of the watershed. Clark's method clearly attempts to relate geomorphic properties to watershed response [Cleveland et al. 2008]. The mathematical form of Clark's instantaneous unit hydrograph (IUH) is represented as:

$$Q_{i+1} = 2C_0 R_{Ei} + C_1 Q_i \tag{3}$$

where:

- i – index varying from 1 to N (N – number of ordinates of the time-area diagram),
- R_{Ei} – uniformly distributed rainfall excess,
- Q_{i+1} – (i+1)-th ordinate of Clark's instantaneous unit hydrograph,
- Q_i – ordinate of the unit hydrograph,
- C_0 and C_1 – weighting coefficients proposed by Muskingham and defined as: $C_0 = 0,5t/(R + 0.5t)$, $C_1 = (R - 0.5t)/(R + 0.5t)$,
- t – computational time interval.

A unit hydrograph for a finite time interval T can be found by lagging IUH equal to time T and averaging the IUH ordinates for the time period T.

The simulations were carried out using HEC-HMS 3.4 software [Hydrologic Modeling System HEC-HMS 2009]. The parameters of Snyder's and Clark's models were determined by optimisation to observe the best agreement of calculated and observed hydrograms. The goal of optimisation is to minimise a scalar quantity known as an objective function or error. The objective function/error may be defined in several ways. The following three objective functions are adopted in this study:

1. The objective function based on minimising the difference between the observed and simulated peak discharges for an event [Ahmad et al. 2009]:

$$F_1 = \frac{(Q_{\mathrm{obs}} - Q_{\mathrm{sim}})^2}{(Q_{\mathrm{obs}})^2} \tag{4}$$

2. The objective function based on least squares method, i.e. on minimising the sum of squares of deviations between the observed and computed values of the runoff hydrograph. Mathematically, this objective function is expressed as [Ahmad et al. 2009]:

$$F_2 = \frac{(Q_{j\mathrm{obs}} - Q_{j\mathrm{sim}})^2}{(Q_{j\mathrm{obs}})^2} \tag{5}$$

3. The objective function defined by Lee et al. [1972] and adopted by Al-Wagdany and Rao [1997] which considers both peak discharge Q_p and time to peak T_p and is defined as follows:

$$F_3 = \left[\left(\frac{Q_{\mathrm{obs}} - Q_{\mathrm{sim}}}{Q_{\mathrm{sim}}}\right)^2 - \left(\frac{T_{\mathrm{pobs}} - T_{\mathrm{psim}}}{T_{\mathrm{psim}}}\right)^2\right]^{\frac{1}{2}} \tag{6}$$

where:

Q_{obs} – observed peak discharge,
Q_{sim} – simulated peak discharge,
$Q_{j\mathrm{obs}}$ – observed value of the j-th ordinate of the direct runoff hydrograph,
$Q_{j\mathrm{sim}}$ – simulated value of the j-th ordinate of the direct runoff hydrograph,
T_{pobs} – time to peak of the observed hydrograph,
T_{psim} – time to peak of the simulated hydrograph.

The coefficient of efficiency E is selected to test the performance of the model as proposed by Nash and Sutcliffe [1970]:

$$E = \left[1 - \frac{\sum_{i=1}^{i=NQ} (Q_{\mathrm{obs}} - Q_{\mathrm{sim}})^2}{\sum_{i=1}^{i=NQ} \left(Q_{\mathrm{obs}} - \overline{Q_{\mathrm{obs}}}\right)^2}\right] \tag{7}$$

where:

- NQ – number of ordinates of the hydrograph,
- i – index varying from 1 to NQ,
- Q_{obs} – i-th ordinate of the observed hydrograph,
- Q_{sim} – i-th ordinate of the simulated hydrograph,
- $\overline{Q_{obs}}$ – mean of the ordinates of the observed hydrograph.

Study area

The Grabinka river is a left-bank tributary of the Wisłoka river (Fig. 1). The drainage basin covers an area of 218.68 km^2, the length of the watercourse is 32.82 km, and the average gradient of the basin, calculated using the Kajetanowicz equation, is 5.46‰. The Grabinka river source is located near the Brzozówka settlement, at about 235 m a.s.l.; it discharges to Wisłoka at about 195 m a.s.l. In the Grabinka drainage basin, there are quaternary formations: sands with rocks, clays and river sands on miocenic loams [Podział hydrograficzny Polski 1983]. High and medium permeability soils prevail in the basin. Most of the terrain is covered in woods and crops. According to the Tarnow station of the Institute of Meteorology and Water Management (IMGW), covering the basin under study, the maximum rainfall of 24 h duration, calculated using Gumbel's method, is 122.7 mm for 1% probability, and 48.9 mm for 50% probability [Dane hydrologiczne… 2009].

Fig. 1. Grabinka river catchment
Rys. 1. Zlewnia rzeki Grabinki

RESULTS AND DISCUSSION

The results on flood wave simulated using Snyder's and Clark's models are shown in Figure 2 against a background of the observed hydrograph. The flood wave under analysis (biggest in 2006) was triggered by rainfall of 83.6 mm which lasted for 120 h. Long-time precipitation causes greatest flood discharges, and as a consequence material damage, in flood areas in the Carpathian Vistula basins [Niedbała and Czulak 2000]. Calculations showed that the *CN* parameter for determining the amount of effective rainfall was 82. Comparing this value with the one acquired using traditional SCS method, which is based on basin usage and soils, we find that it is in line with the 3rd degree of moisture. This is caused by the fact that the flood wave under analysis was preceded by a lower wave with culmination of 5.79 $m^3 \cdot s^{-1}$, which happened 108 h earlier and was triggered by 35 mm rainfall lasting for 96 h. The volume of direct runoff was $5.78 \cdot 10^6$ m^3, which gives runoff of 26.4 mm. The shape of waves calculated using SUH is close to the observed wave. The times to culmination of SUH are the same as for the observed wave. The value of Snyder's SUH culmination discharge is 0.11% higher than the observed one, while that of Clark's SUH is 1.9% lower.

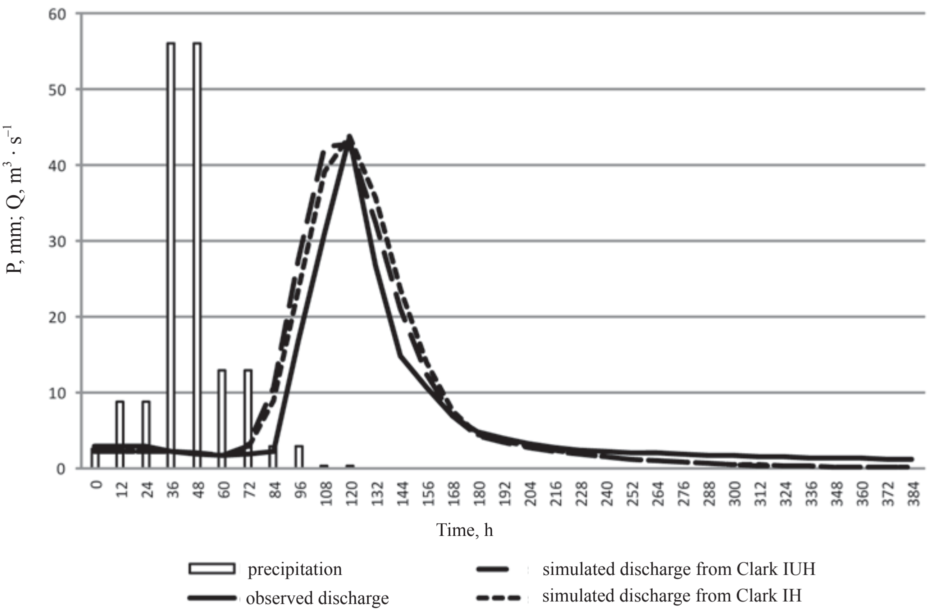

Fig. 2. Comparison of observed and SUH hydrographs for case study (P – precipitation, Q – discharge)

Rys. 2. Porównanie hydrogramu obserwowanego i SUH na przykładzie analizowanego epizodu (P – opad, Q – przepływ)

The difference in flood total volume between the observed episode and Snyder's SUH was 11.7%, and for Clark's SUH it was 11.6%. After the calibration of model parameters values it turned out that for Snyder's SUH the optimal T_{lag} value was 30 h and C_p = 0.80, while for Clark's SUH the time of concentration T_c was 34 h and the storage coefficient R = 15 h. For those values we received the lowest values of the objective functions (Table 1).

Table 1. SUH model performance at calibration using different error measures
Tabela 1. Przebieg kalibracji SUH przy wykorzystaniu różnych miar błędu

Objective function Funkcja celu	Snyder UH T_{lag} = 30 h; C_p = 0.80	Clark UH T_c = 34 h; R = 15 h
F_1	$1.206 \cdot 10^{-6}$	0.000358
F_2	0.0775	0.0916
F_3	0.0011	0.0193

Lower values of the objective function were calculated using models based on culmination flow analysis, i.e. F_1 and F_3, compared to model F_2 in which the whole hydrograph is used. An example graph of the objective function for T_{lag} and F_1 model is shown in Figure 3.

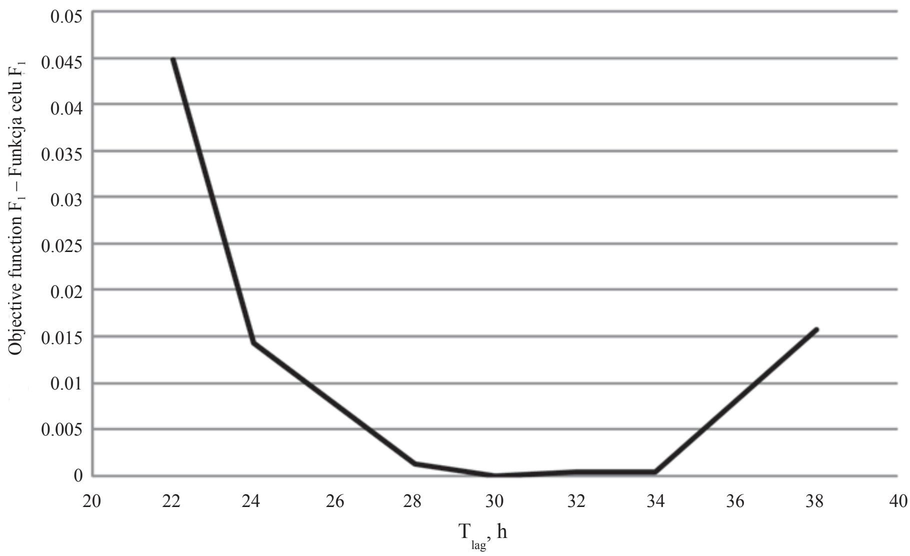

Fig. 3. Optimum T_{lag} yielded by different values of objective function F_1
Rys. 3. Optymalizacja wartości T_{lag} dla różnych wartości funkcji F_1

Similar observations were made by Ahmad et al. [2009] testing Clark's model in the Kaha river basin in Pakistan. For T_{lag} equal to 30 h, the value of C_t coefficient was 4.671, which is much higher than the one given by Snyder (C_t from 1.35 to 1.65) [Ponce 1989] and the one reported by Belete [2009] for the Awash and Tekeze basin in Ethiopi (C_t between 0.362 and 0.736). The latter author states that according to many researchers the C_t value varies in a wide range of 0.3 to 6.0, and that its great variability depends on local conditions. In this research the C_t value was 0.8, which fits well in the above-mentioned range.

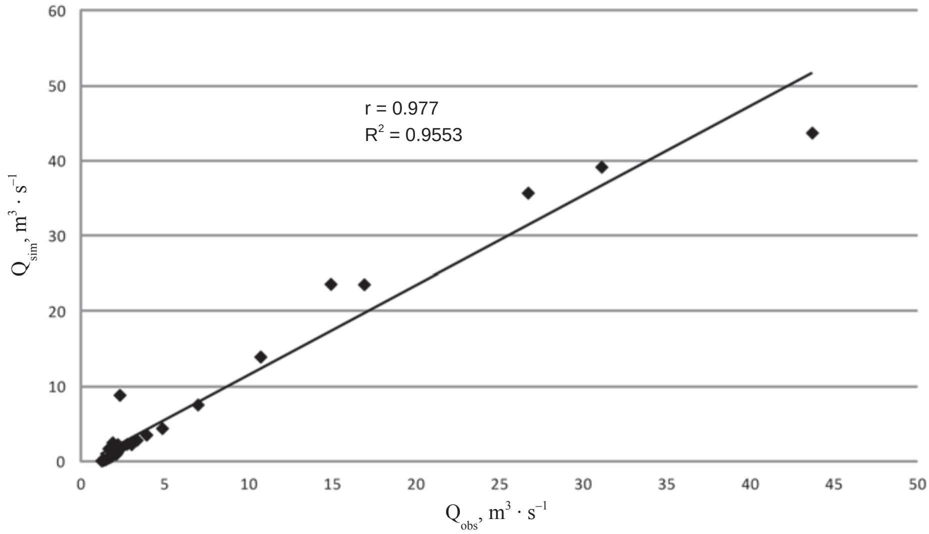

Fig. 4. Observed versus Snyder UH-simulated flows
Rys. 4. Zależność między przepływami obserwowanymi a obliczonymi według modelu Snydera

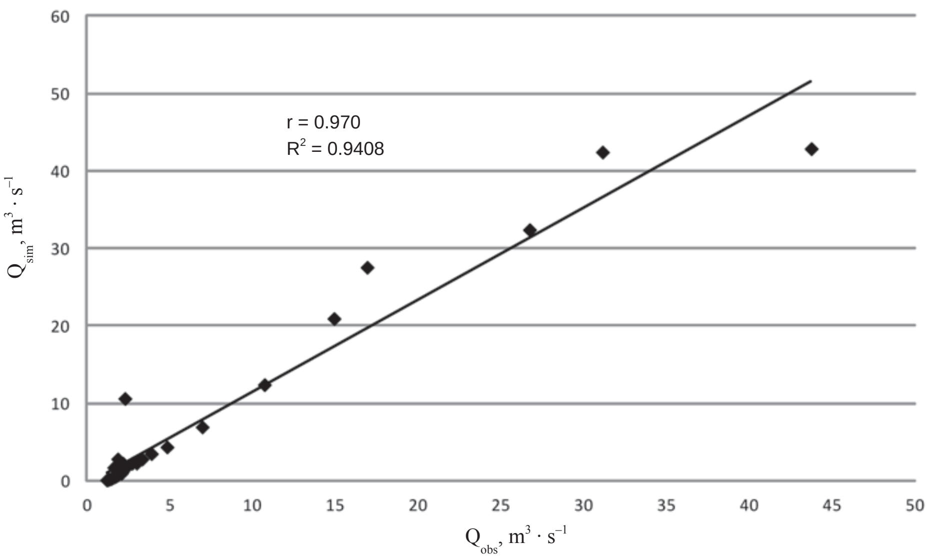

Fig. 5. Observed versus Clark UH-simulated flows
Rys. 5. Zależność między przepływami obserwowanymi a obliczonymi według modelu Clarka

Figures 4 and 5, showing high values of the coefficient of correlation r and the coefficient of determination R^2, attest the correctness of hydrograph simulation. For the hydrograph calculated using Snyder's model the coefficient of determination is equal to 0.955, and is slightly higher than the one calculated with Clark's model (R^2 = 0.941).

The calculated coefficients of correlation for Snyder's and Clark's model are statistically significant at α = 0.05. The value of t-test is 22.188 for Clark's model and 25.740 for Snyder's model. In both cases, the best results of simulation were obtained for lowest discharge flows. When the discharge flow rises, the data points in Figures 3 and 4 tend to deviate from linear regression. The quality of both models was also assessed on the basis of the efficiency coefficient *E*. Its values, 89% for Snyder's SUH model and 87% for Clark's SUH, confirm successful simulation of flood discharges in the Grabinka river basin using SUH.

CONCLUSION

In the course of analysis we confirmed the suitability of Snyder's and Clark's models to simulate flood discharges in the Grabinka river basin, with slightly better results being obtained using the former model. The efficiency coefficient values of 89% for Snyder's SUH model and 87% for Clark's SUH model seem to attest it. For both SUH the times to culmination were the same as the observed one, while the culmination flow discharge was 0.11% higher in Snyder's model and 1.9% lower in Clark's model, compared to the observed discharge. For limited data the objective function based on peak values of discharge gives better results than the objective function based on the complete hydrograph. To confirm the correctness of the results obtained in this study it is necessary to continue research on greater data set. Snyder's model, owing to the limited number of parameters it comprises and a relative ease of their acquirement, shall be especially recommended for practical use.

REFERENCES

Ahmad M.M., Ghumman A.R., Ahmad S., 2009. Estimation of Clark's instantaneous unit hydrograph parameters and development of direct surface runoff hydrograph. Water Resour. Manage., doi:10.107/s11269-008-9388-8.

Al-Wagdany A.S., Rao A.R., 1997. Estimation of the velocity parameter of the geomorphologic instantaneous unit hydrograph. J. Water Resour. Manage. 11(1), 1–16, doi:10.1023/A:1007923906214.

Belete M.A., 2009. Synthetic unit hydrographs in the Upper Awash and Tekeze basins. Methods, procedures and models. VDM Verlag Dr Müller.

Clark C.O., 1945. Storage and the unit hydrograph. Trans. ASCE 110, 1419–1446.

Cleveland T.G., Thompson D.B., Xing F., Xin H., 2008. Synthesis of unit hydrographs from a digital elevation model. J. Irrig. Drain. Eng. 134(2), 212–221.

Dane hydrologiczne pozyskane na potrzeby projektu „Określenie zagrożenia powodziowego w zlewni Wisłoki", 2009. IMGW Kraków.

Hydrologic Modeling System HEC-HMS, 2009. User's Manual. U.S. Army Corps of Engineers, Hydrologic Engineering Center, Davis.

Kundzewicz Z.W., 2001. Non-structural flood protection and sustainability. [In:] Non-structural measures for water management problems. Ed. S.P. Simonovic. Proceedings of the International Workhsop. IHP-V Technical Documents in Hydrology No. 56, UNESCO Paris, 8–27.

Lee M.T., Blank D., Delleur J.W., 1972. A program for estimating runoff from Indiana watershed. Part II. Assembly of hydrologic and geomorphologic data for small watersheds in Indiana. Tech. Rep. No. 23. Purdue University Water Resources Research Center, Lafayette.

Limantara L.M., 2009. The limiting physical parameters of synthetic unit hydrograph. World Appl. Sci. J. 7(6), 802–804.
Nandalal K.D.W., 2009. Use of a hydrodynamic model to forecast foods of Kalu River in Sri Lanka. J. Flood Risk Manage. 2, 151–158.
Nash J.E., Sutcliffe J.V., 1970. River flow forecasting through conceptual models. Part I. A discussion of principles. J. Hydrol. (Amst.) 10(3), 282–290, doi:10.1016/0022-1694(70)90255-6.
Niedbała J., Czulak J., 2000. Warunki i scenariusze hydrologiczne wezbrań. [In:] Model kompleksowej ochrony przed powodzią w obszarze dorzecza górnej Wisły na przykładzie woj. małopolskiego. Ed. M. Maciejewski. IMGW Kraków, 93–126.
Podział hydrograficzny Polski. Cz. I, 1989. IMGW Warszawa.
Ponce V.M., 1989. Engineering hydrology: Principles and practices. Prentice Hall Upper Saddle River, N.J.
Salami A.W., Bilewu S.O., Ayanshola A.M., Oritola S.F., 2009. Evaluation of synthetic unit hydrograph methods for the development of design storm hydrographs for some rivers in South-West, Nigeria. J. Am. Sci. 5(4), 23–32.
Soczyńska U., 1997. Hydrologia dynamiczna. Wyd. Nauk. PWN, Warszawa.
Soczyńska U., Gutry-Korycka M., Buza J., 2003. Ocena zdolności retencyjnej zlewni. [In:] Rola retencji zlewni w kształtowaniu wezbrań opadowych. UW Warszawa, 77–104.
Straub T.D., Melching C.S., Kocher K.E., 2000. Equations for estimating Clark unit hydrograph parameters for small rural catchments in Illinois. Water-Resources Investigations Report 00-4184 USGS.

WYKORZYSTANIE HYDROGRAMU JEDNOSTKOWEGO SNYDERA I CLARKA DO OBLICZEŃ FAL POWODZIOWYCH W ZLEWNI WYŻYNNEJ (NA PRZYKŁADZIE RZEKI GRABINKI)

Streszczenie. Na przykładzie wyżynnej zlewni Grabinki zlokalizowanej w dorzeczu Wisłoki oceniono możliwości zastosowania syntetycznego hydrogramu jednostkowego (SUH) Snydera i Clarka do symulacji wezbrań powodziowych. Kalibrację parametrów modeli oparto na zanotowanym epizodzie opadowym z czerwca 2006 r. Jako kryterium optymalizacji przyjęto minimum funkcji celu. Jakość modeli oceniono za pomocą współczynnika efektywności *E*. Analiza wykazała, że SUH Snydera i Clarka dobrze opisywały falę rzeczywistą, przy czym nieco lepsze wyniki uzyskano dla pierwszego modelu. W przypadku obu SUH uzyskano takie same czasy do kulminacji jak dla fali rzeczywistej, natomiast obliczony przepływ w kulminacji różnił się od obserwowanego – w przypadku użycia modelu Snydera był wyższy o 0,11%, a w przypadku modelu Clarka niższy o 1,9% w stosunku do przepływu obserwowanego.

Słowa kluczowe: syntetyczny hydrogram jednostkowy, funkcja celu, optymalizacja

Accepted for print – Zaakceptowano do druku: 16.02.2011.