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STATISTICAL METHODS FOR TREND INVESTIGATION IN HYDROLOGICAL NON-SEASONAL SERIES

METODY STATYSTYCZNE BADANIA TRENDU W NIESEZONOWYCH SZEREGACH HYDROLOGICZNYCH

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Abstract. In the paper some methods for identification of the monotonic deterministic trend in hydrological time series were presented and discussed: the *t*-test, the Mann-Kendall test with corrections, the Theil-Sen estimator with the test based on bootstrap and the *pre-whitening* method. The tests varied in assumptions on the existence of the serial correlation and the distribution of the variables in the series. Occasionally, when assumptions were not fulfilled, the tests led to contradictory conclusions. As examples, the applications to annual maximum and minimum flows were presented.

Streszczenie. W artykule zaprezentowano metody statystyczne wykrywania deterministycznego, monotonicznego trendu w szeregach hydrologicznych: test *t*, test Manna-Kendalla wraz z poprawkami, estymator Theila-Sena wraz z testem opartym na metodzie bootstrap oraz metodę *pre-whitening*. Testy różniły się założeniami dotyczącymi autokorelacji i funkcji rozkładu badanych zmiennych. Niekiedy, jeśli założenia testów nie były spełnione, zastosowanie ich doprowadziło do sprzecznych wniosków. W przykładach zastosowano te testy do maksymalnych i minimalnych przepływów rocznych.

Key words: hydrological series, monotonic trend, statistical test

Słowa kluczowe: szereg hydrologiczny, trend monotoniczny, test statystyczny

INTRODUCTION

Deterministic monotonic trend in hydrological series is a result of gradual natural or anthropogenic changes in a watershed and is the source of non-homogeneity. Its identification allows an insight into the structure of the series and the forecast of its future

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behavior. Among classical methods, the *t*-test, the Mann-Kendall test, the Theil-Sen slope estimator and the Spearman's rho test are the most important. The basic assumption for the *t*-test is normality of the variables in the series, what is mostly not fulfilled in hydrology. In turn, the existence of the serial or seasonal correlation in hydrological series excludes the usage of the original version of the Mann-Kendall test which assumes independence of the successive terms. In such a case a correction of the classical version of the Mann-Kendall test should be applied.

The literature provides many studies on statistical methods for trend identification in hydrological time series. The problem is complex because the difficulty lies in distinguishing between natural variability and trends [Askew 1987] and in respecting the influence of autocorrelation on methods for the detection of trends [Yue et al. 2002]. Examples of studies on trends are: Lettenmaier [1976], Hirsch and Slack [1984], McLeod and Hipel [1994], Svensson et al. [2005], Khaliq et al. [2006], Mudelsee et al. [2006]. Among polish authors Kundzewicz et al. [2004, 2005] investigated trends in annual maximum flow and Węglarczyk [2009] discussed the detection of trends in annual lake levels.

The main aim of this paper was to present, characterize and compare statistical methods for trend investigation in hydrological series. The effectiveness of the methods for identification of the monotonic behavior was justified. As examples, one polish, one canadian and one american rivers were considered.

MATERIAL

In the paper the following hydrologic series were investigated:

- the maximum and minimum annual discharges on the Red River at Ste. Agathe, 1959–2010,
- the maximum and minimum annual discharges on the Nida River at Pińczów, 1951–2011,
- the maximum annual discharges on the Arkansas River at Canon City, 1951–2011.

The source of the the Nida River data was the Institute of Meteorology and Water Management, Poland, of the Red River data was Environment Canada and of the Arkansas River was U.S. Geological Survey (National Water Information System).

To obtain the preliminary insight into the series, the time series plot and autocorrelation functions had to be observed. The time series plot may suggest the existence of a trend but it does not decide definitely on the tendency to changes. As an doubtful example the plot for the Red River at Ste. Agathe was presented in Figure 1.

The shapes and values of the autocorrelation functions have an influence on the choice of the method for trend investigation. The discharges on the Nida River and on the Red River were featured by a significant autocorrelation. The autocorrelation function (ACF) and partially autocorrelation function (PACF) were significant for the Red River at the first lag in minimum annual flows and at the sixth lag in maximum annual flows. The ACF and PACF were significant for the maximum and minimum flows on the Nida River at the first lag. The graphs of the ACF and the PACF for the Red River were presented in Figures 2 and 3. The ACF and PACF were not significant for the Arkansas River and the discharges could be treated as independent.

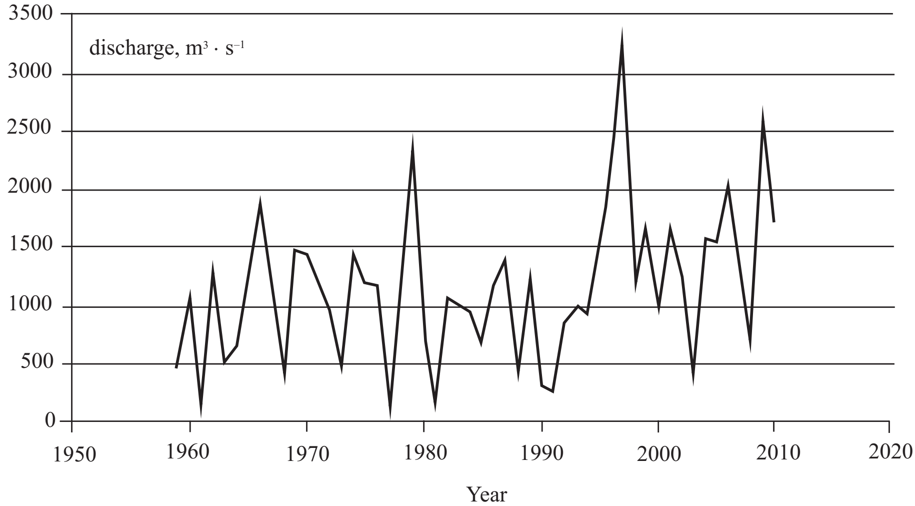

Fig. 1. The maximum annual flows, the Red River
Ryc. 1. Przepływy maksymalne roczne, Red River

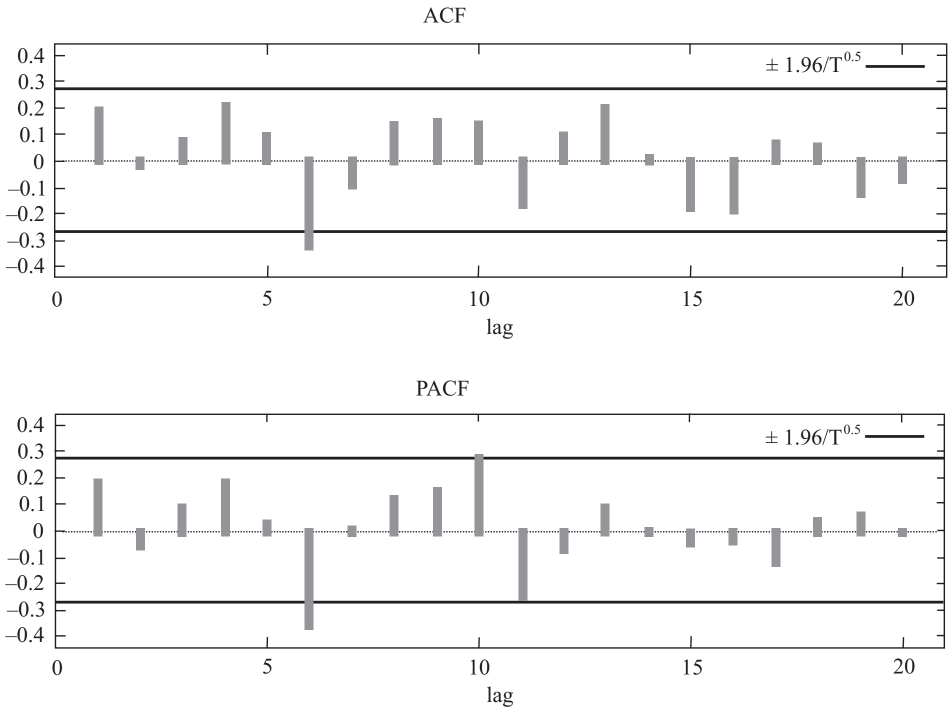

Fig. 2. The autocorrelation functions of the maximum annual flow, the Red River
Ryc. 2. Funkcje autokorelacji przepływów maksymalnych rocznych, Red River

All variables were characterized by a right-skewed distribution function. The skewness in the samples varied from 0.54 for the minimum flows on the Red River to 2.03 for the minimum flows on the Nida River.

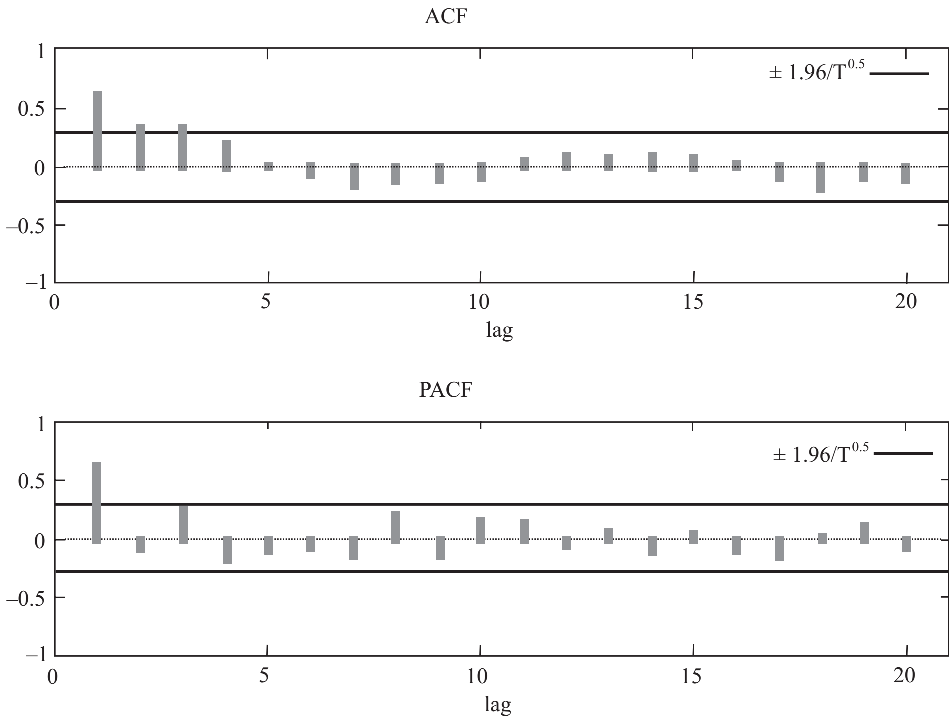

Fig. 3. The autocorrelation functions of the minimum annual flow, the Red River
Ryc. 3. Funkcje autokorelacji przepływów minimalnych rocznych, Red River

METHODS

The basic model for hydrological time series considered in this paper was:

$$X_t = X_t^{Det} + Z_t, \tag{1}$$

where:

$t \in T = \{1, 2, \ldots\}$,
Z_t – a stochastic term,
X_t^{Det} – a deterministic component which generally may contain monotonic trend X_t^{Tr}, step trend, seasonal pattern or another function of t .

The important task in stochastic hydrological modeling is to specify the both components properly. In this paper the deterministic part as monotonic trend X_t^{Tr} was discussed.

The statistical hypotheses in testing for trend were:

H: *No monotonic trend exists in the series*,

H_A: *The series is characterized by a monotonic trend*.

Various statistical tests are available to verify above hypotheses. In this paper the parametric and nonparametric methods were applied. The one-sided versions of the tests were considered in this paper.

Parametric method

The parametric model was given as $X_t^{Tr} = f(t)$, where *f* is generally a monotonic function: polynomial, exponential, logarithmic etc., and the parameters of *f* are estimated by the Least Square Error Method. In this article the simplest, linear form was applied, $X_t^{Tr} = a + bt$. The estimate of *b* is given by:

$$b = r_P \frac{S_x}{S_Y} \tag{2}$$

where:

r_P – the Pearson's correlation coefficient,
S_X, S_Y – the standard deviations in the samples.

The *t* – test verifies the hypotheses:

H: $b = 0$, H_A: $b > 0$ ($b < 0$).

In this paper the choice of the inequality in the hypothesis H_A was related to the increasing or decreasing character of changes.

The model for the *t* – test is based on the assumption that the variables X_t in (1) are independent and normally distributed. In hydrological series this opinion is usually unrealistic, because distributions are skewed and variables are often serially correlated. For skewed distributions the *t* – test has a low power comparing to the normal case. Moreover, the size of the test may exceed the fixed significance level. Sometimes the Box-Cox transformation is useful to achieve normality but this problem is beyond the scope of this paper.

No one variable considered in this paper satisfied condition about normality. The application of the *t* – test in this paper was carried out to present how a wrong decision can be obtained.

Nonparametric methods

In these methods no prior knowledge about the shape of dependence is specified and the tests are characterized by a higher power than parametric tests, when distributions are not symmetric.

a. the Theil-Sen slope estimator [Theil 1950, Sen 1968] of the parameter *b* is defined as:

$$ß = med \frac{X_j - X_i}{j - i} \text{ for } j > i, \text{ where } i, j \in T. \tag{3}$$

It is based on the median and may be used if the distribution of X is skewed. Additionally, ß is insensitive to outliers. The basic hypotheses are:

H: *The slope is zero*, H_A: *The slope is greater* (*less*) *than zero*

The test for the Theil-Sen slope is built on the bootstrap method. In the bootstrap method, M samples, each with n elements, are drawn with replacement from the original sample and, using the equation (3), the slope for each sample is computed. Then estimates $ß_1, ß_2, \ldots, ß_M$ are set in an ascending order. Subsequently, the empirical cumulative distribution function F_n is obtained. The *p-value* for the statistic ß is calculated as:

$$p\text{-}value = F_n(\beta) = \frac{k_\beta}{M}, \tag{4}$$

where:

$k_ß$ – the number of estimates which are less or equal ß, $k_\beta = \sum_{\beta_i \le \beta} 1$.

If no trend exists in the series, the *p-value* should be approximately 0.5 in the bootstrap method. The trend is positive if *p-value* is greater than 1-α and negative if it is less than α, where α is the significance level of the test. The location of ß is determined in relation to percentiles for which the empirical cumulative distribution function equals α or 1-α. The number of bootstrap samples was chosen as 1000 in this paper, as suggested in Efron and Tibshirani [1993].

b. The Mann-Kendall test [Kendall 1938, Mann 1945] is the most often used test among nonparametric methods for detection of the monotonic trend. The MK statistic has the form

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn\left(x_j - x_i\right).$$

The variance of S equals $Var(S) = \frac{n(n-1)(2n+5)}{18}$. The test statistic:

$$U = \begin{cases} \frac{S-1}{\sqrt{Var(S)}} & if\ S > 0 \\ 0 & if\ S = 0 \\ \frac{S+1}{\sqrt{Var(S)}} & if\ S < 0 \end{cases}$$

follows the standardized normal distribution. If U is significantly less than 0 (greater than 0), then the decreasing (increasing) trend exists in the series.

The basic assumption for this test is the lack of autocorrelation. If this assumption is not fulfilled then the variance *Var*(*S*) is underestimated. The effective number of independent samples of a serially correlated series is less than the actual number of samples.

Therefore, a positive serial correlation increases the probability that the Mann-Kendall test specifies the trend as significant, although in reality no trend exists. In other words, serial correlation increases the type I error. That's why, for serially dependent series, Bayley and Hammersley [1946] suggested the correction of the variance to $Var^*(S) = \frac{n}{n^*} Var(S)$ using "effective number of observations" as:

$$n^*_{BH} = \frac{n}{1 + \frac{2}{n}\sum_{j=1}^{n-1}(n-j)\rho_j}, \tag{5}$$

where:

ρ_j – the autocorrelation coefficient of *j*-th order.

In the next formula [Hamed and Rao 1998] the effective number of observations was proposed as:

$$n^*_{HR} = \frac{n}{1 + \frac{2}{n(n-1)(n-2)}\sum_{i=1}^{n-1}(n-i)(n-i-1)(n-i-2)\rho_S(i)}, \tag{6}$$

where:

ρ_S – the autocorrelation function of ranks of observations.

In the third correction [Matalas and Langbein 1962] the effective sample size is:

$$n^*_{ML} = \frac{n}{1 + 2\frac{\rho_1^{n+1} - n\rho_1^2 + (n-1)\rho_1}{n(\rho_1 - 1)^2}} \tag{7}$$

where:

ρ_1 – the autocorrelation of the 1st order. This correction is used if the process is assumed to be AR(1).

c. The Spearman rank correlation coefficient ρ_S is the Person correlation coefficient applied to the ranks R_1, R_2, …, R_n and the series 1, 2,…, *n*. This method also requires independency.

d. The *pre-whitening* method [Storch 1995] is given as $y_t = x_t - \rho_1 x_{t-1}$.

This formula is applied to reduce the influence of an AR(1) component on the Mann Kendall test, but it is insufficient if the process is of higher order. *Pre-whitening* sometimes causes the inaccurate reduction of the magnitude of the existing trend [Yue et al. 2002].

The significance levels were considered as 0.05 and 0.01 in this paper.

RESULTS

The results for investigation of the trend of the maximum and minimum flows were collected in Table 1. Observe that for each series, except for the Nida River (minimum flows), the *p-value* of the *t*-test for slope was not less than that of the Mann-Kendall test.

The final decision on accepting or rejecting the null hypothesis were dependend on the method of testing and on the significance level. For the Red River and the Nida River the Mann-Kendall test with corrections and the Theil-Sen test were decisive. For the Arkansas River the Mann-Kendall test (without correction), the Theil-Sen slope and the Spearman test allowed to formulate the final conclusion.

Case 1: $\alpha = 0.05$

The maximum and minimum discharges on the Red River and maximum discharges on the Nida River exhibited the trend behavior with probability 0.95. No trend was observed in minimum annual flows on the Nida River. For the Arkansas River the results differed: the Mann-Kendall and the Spearman tests indicated trend while the bootstrap test for the Theil-Sen slope did not. In this case we relied on the Mann-Kendall which has a greater power than the Theil-Sen test. The *pre-whitening* method caused the increase of the *p-values* but the final decision remained unchanged.

Case 2: $\alpha = 0.01$

No trend existed except for the Nida River minimum discharges. For this significance level the following influence of the autocorrelation on the Mann-Kendall test was observed: when the test was applied to the minimum Red River discharges and maximum Nida River discharges then the *p-value* was less than 0.01 which recommended the decision about trend, on the other hand the corrections indicated the acceptance of the null hypothesis. The final decision was took up on the basis of the corrected version of the Mann-Kendall test. The *t*-test indicated trend for the Red River and the Nida River maxima which was wrong because the Mann-Kendall (with corrections) did not. This was a consequence of the positive skewness of the empirical distribution functions.

CONCLUSION

The presented tests and examples justified that statistics provides many various methods for trend investigation. Each method, however, should be applied with caution according to theoretical assumptions. Application of statistical methods and practical experience complement each other and are effective tools in hydrological modeling.

Table 1. The *p-values* for the tests for trend
Tabela 1. Wartości *p-value* testu na trend

Series \ test Szereg \ test	*t*-test for slope *b*	Theil-Sen slope	MK	MK_{BH}	MK_{HR}	MK_{ML}	Spearman	Pre-whitening	Conclusion Wniosek α = 0.05	Conclusion Wniosek α = 0.01
Maximum annual flows, the Red R. Przepływy maksymalne roczne, Red R.	0.004	0.013	0.016	0.027	0.042	×	×	×	↑	0
Minimum annual flows, the Red R. Przepływy minimalne roczne, Red R.	0	0.040	0.004	0.035	0.152	0.014	×	*p-value*, *t*-test = 0.003 *p-value*, MK test = 0.007	↑	0
Maximum annual flows, the Nida R. Przepływy maksymalne roczne, Nida	0.006	0.008	0.006	0.030	0.088	×	×	×	↓	0
Minimum annual flows, the Nida R. Przepływy minimalne roczne, Nida	0.219	0.256	0.197	0.181	0.069	0.726	×	*p-value*, *t*-test = 0.238 *p-value*, MK test = 0.207	0	0
Maximum annual flows, the Arkansas R. Przepływy maksymalne roczne, Arkansas	0.026	0.079	0.035	×	×	×	0.029	×	↓	0

× — the test was not applied because assumptions were not fulfilled – nie zastosowano testu z powodu niespełniania założeń; ↑ (↓) – increasing (decreasing) trend – trend rosnący (malejący) ; 0 – no trend – brak trendu.

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