ACTA SCIENTIARUM POLONORUM *Acta Sci. Pol., Formatio Circumiectus 13 (2) 2014, 23–37*

# ON MEAN TEMPERATURE IN CLIMATOLOGIC AND AGRONOMIC STUDIES

# O TEMPERATURZE ŚREDNIEJ W STUDIACH KLIMATOLOGICZNYCH I AGRONOMICZNYCH

Wiesław Kowalski
University of Agriculture in Krakow

**Abstract.** The paper discusses various methods of determining mean temperature in climate studies and the consequences of applying particular methods. The conclusion is that the use of various mathematical formulas that describe mean temperature usually yields different results. In some cases, the differences can reach ten to twenty percent. Consequently, the results of different analyses are incomparable, even in engineering, and especially in research and scientific studies. Hence the postulate to avoid conducting comparative analyses with the use of data from different authors, if their temperature definitions are different, or if there is no precise information on the formulas used in the calculation.

**Streszczenie.** W pracy przedstawiono rozważania na temat sposobów wyznaczania temperatury średniej w analizach klimatologicznych oraz skutków stosowania poszczególnych sposobów. Z rozważań tych wynika wniosek, iż stosując różne formuły matematyczne, opisujące temperaturę średnią, zwykle uzyskuje się różne wartości. W pewnych sytuacjach różnice mogą sięgać kilkudziesięciu procent. Uzyskane tą drogą wyniki analiz są zatem nieporównywalne, nawet w zastosowaniach inżynierskich, a tym bardziej w pracach o charakterze naukowo-badawczym. Stąd postulat, aby w pracach naukowych unikać prowadzenia analiz porównawczych z wykorzystaniem danych pochodzących od różnych autorów, jeżeli stosowane przez tych autorów definicje temperatury średniej są różne lub jeżeli brak jest precyzyjnych informacji o stosowanych formułach obliczeniowych.

**Key words:** mean temperature, sum of active temperatures, comparative analysis, Nyquist's theorem

**Słowa kluczowe:** temperatura średnia, suma temperatur aktywnych, analiza porównawcza, twierdzenie Nyquista

---

Correspondinb author – Adres do korespondencji: dr inż. arch. Wiesław Kowalski, Katedra Budownictwa Wiejskiego Uniwersytet Rolniczy im. Hugona Kołłątaja w Krakowie, al. Mickiewicza 24/28, 31-120 Kraków, e-mail: wkowalski@ur.krakow.pl.

## INTRODUCTION

The assessment of climatic conditions for the cultivation of particular plants takes into account thermal conditions characteristic for the region, e.g. mean temperature (daily, monthly, etc.). Sometimes, other agro-climatic parameters are used, such as *sum of active temperatures* (SAT), a very popular parameter with wine grape growers. However, also in this case what most significantly affects the outcome of the assessment is daily mean temperature, which influences the calculated SAT. Thus, it is very often the case that agricultural climatology or agronomy poses a question about the determination of mean temperature. Yet, the analysts are not always aware of the fact that the way in which mean temperature is calculated may, in some cases, significantly affect the calculated values. Consequently, the results obtained by different researchers may be unsuitable for comparative analyses. Sometimes such analyses concern historical data, obtained few dozen or even a few hundred years ago. This data is not always fully described in terms of measurement methodology and applied definition of mean temperature. It is therefore desirable that researchers are aware of errors resulting from these situations.

## CALCULATING MEAN VALUES

Calculating mean values of a continuous process, such as the temperature curve, requires, in principle, the use of Riemann integral with the limits of integration set at the beginning and the end of the process ($t_{min}$ and $t_{max}$).

$$T_{mean} = \frac{1}{(t_{max} - t_{min})} \int_{t_{min}}^{t_{max}} T(\tau)\,d\tau \tag{1}$$

So the resulting "mean" is interpreted as "the effective value". Rarely however, does modern science use analog tools to conduct continuous measurements, also in the case of meteorological processes. Readings and registration are usually conducted at specified intervals, such as once a day, every hour, every 15 minutes, etc. Consequently, a continuous process is modeled with a finite and countable set of numbers that represent this process. In this situation, the process of calculating mean values can be reinterpreted.

There are at least a few well-known and popular formulas for the concept of "mean" in a discrete set of numbers. It can be understood as mathematical mean, mathematical "weighted" mean, geometric mean or otherwise. If the measurements are made every hour, daily mean will encompass 24 values; in the case of measurements conducted every fifteen minutes – 96 temperature values, etc.

$$T_{mean} = \frac{\sum_{k=1}^{n} T(t_k)}{n} \tag{2}$$

where:
$T(t_k)$ – temperature measured at a moment of time $t_k$,
$n$ – number of samples per day.

"Mean temperature" is very often determined on the basis of only two recorded values: maximum and minimum temperature (within a day):

$$T_{mean} = \frac{1}{2}(T_{max} + T_{min}) \tag{3}$$

or two temperature values recorded at particular times of day, for example at 5:00 am (assumed time of minimum temperature) and at 2 pm (assumed time of maximum temperature):

$$T_{mean} = \frac{1}{2}[T(t_1) + \mathrm{T}(t_2)] \tag{4}$$

It is also possible to apply weighted average, with regards both examples above:

$$T_{mean} = \alpha \cdot T(t_1) + \beta \cdot T(t_2) \tag{5}$$

assuming that: $\alpha + \beta = 1$
where:
$\alpha, \beta$ – weighs attributed to measured temperature values.

In the case of a continuous process, determining the mean according to formula (2) is unjustified because the set of process values (temperature in our case) is unlimited and uncountable.

## MEAN VALUES DETERMINED WITH VARIOUS METHODS

As mentioned above, the term "mean value" is not unambiguous. The mean can be determined in many ways, and in fact infinitely many, given the fact that "weights" in the case of weighted mean can be set freely, as long as their sum is equal to one unit. To simplify this discussion, let us assume that we will primarily take into consideration mean values determined with the formulas (1) to (3), which in practice are used very often. The question arises then: under what circumstances can the application of different mathematical formulas lead to equal mean values?

Is it possible that mean values obtained with the help of formulas (1) and (3) are equal? The answer to this question must be "yes". Such a case, for example, occurs when the temperature course in the studied period is antisymmetric, as shown in Fig. 1. Antisymmetry is understood as a situation in which:

$$\exists t_0 := {}^1/_2 \cdot (t_{min} + t_{max}) : \forall t : (t_0 - t) \in \langle t_{min}, t_{max} \rangle \wedge (t_0 + t) \in \langle t_{min}, t_{max} \rangle : T(t_0 - t) = \\ = -T(t_0 + t)$$

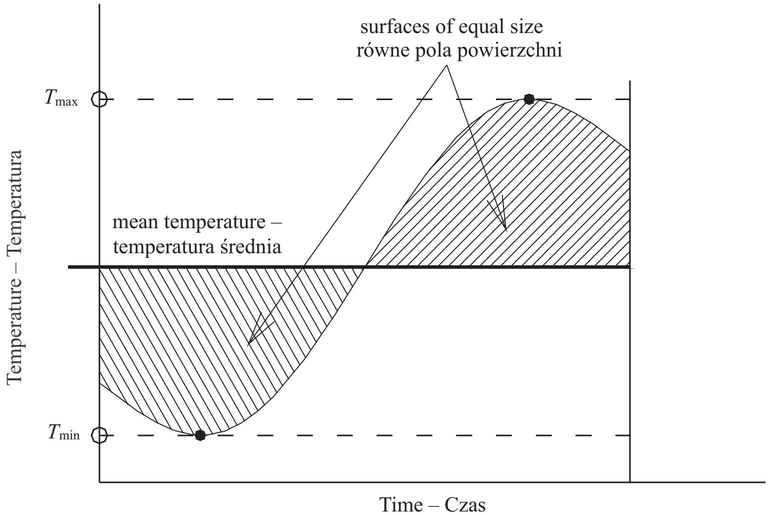


Fig. 1. Exemplary situation when mean daily temperatures, determined with formulas (1) and (3) are equal

Ryc. 1. Przykład sytuacji, gdy temperatury średnie dobowe, wyznaczone ze wzorów (1) oraz (3) są równe

The geometric center between the minimum and the maximum values (i.e. the result of applying the formula (3)) defines a straight line $T = const$ dividing the temperature chart into two parts of equal surface area (this is the effective temperature value determined by the formula (1)).

Antisymmetric temperature course is only an example of such a situation here. The actual chart does not have to be antisymmetric, but it should be close to the harmonic process, because only then the arithmetic mean between the peaks values divides the chart into parts of equal size. Actual temperature courses rarely have such characteristics in real life. Only in deeper soil layers, due to thermal inertia of rock and water, temperature courses have almost harmonic shapes, and mean values from the formulas (1) and (3) are comparable. Fig. 2 illustrates such a situation.

It may be possible that the mean values obtained with formulas (1) and (4) are equal. For example, this occurs when the temperature chart over the given period has harmonic characteristics and the temperature reading time points according to the formula (4) are spaced apart by half of the harmonic process cycle, namely that:

$$t_2 - t_1 = \frac{1}{2} t_0 = \frac{1}{2 \cdot f_0} \tag{6}$$

where:

$t_0$ – harmonic process cycle,
$f_0$ – process frequency.

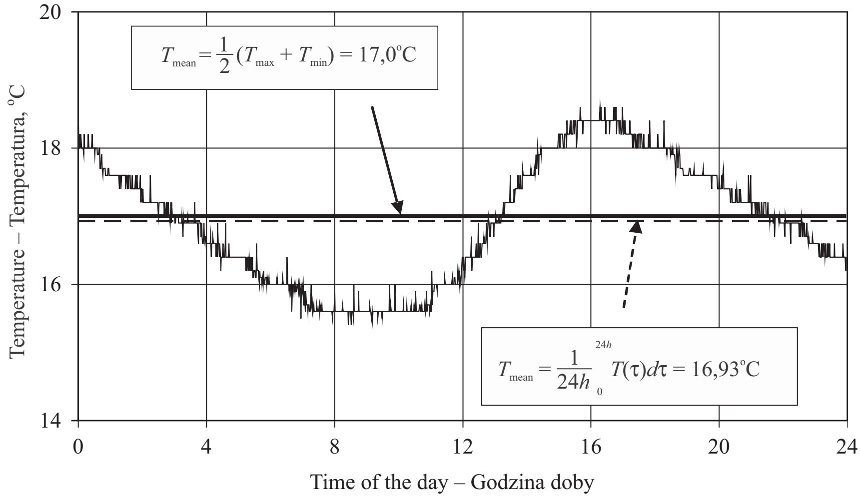


Fig. 2. Daily mean temperature values of soil 25 cm below the ground level on 30 April 2011 (based on continuous measurement in the area artificially devoid of vegetation)
Ryc. 2. Wartości temperatury średniej dobowej gruntu 25 cm pod powierzchnią terenu 30 kwietnia 2011 r. (na podstawie pomiaru ciągłego w terenie sztucznie pozbawionym roślinności)

Such a situation has been shown in Fig. 3. A similar situation occurs when the points $t_1$ and $t_2$ are positioned symmetrically with respect to half of the harmonic process cycle, i.e.:

$$t_2 = t_0 - t_1 \qquad \text{or} \qquad t_1 = t_0 - t_2 \tag{7}$$

Obtained temperature values $T(t_1)$ and $T(t_2)$ do not necessarily correspond to the extreme values of temperature to have their geometric center set a straight line marked as $T = const$, dividing the temperature chart into surfaces of equal size, as in the previous example. Moreover, in this case (that is, for the harmonic temperature course) three "approaches" to mean value are equal: according to the formulas (1), (3) and (4).

As already mentioned, daily temperature courses rarely exhibit harmonic characteristics. This is because temperature variations in the environment (air, soil) are caused by overlapping effects of various phenomena, such as solar radiation, precipitation, evaporation and condensation of water, cold air drains, etc.; and each of them is a dynamic phenomenon not necessarily periodic. Actual temperature courses are changeable in the time, but they are not purely harmonic processes. Hence the difficulty in choosing appropriate time points $t_1$ and $t_2$ for the correct application of formula (4) in determining mean value. Certainly, these time points exist but they are different every day, so it is difficult to determine the mean value according to the mentioned formula in an unambiguous way.

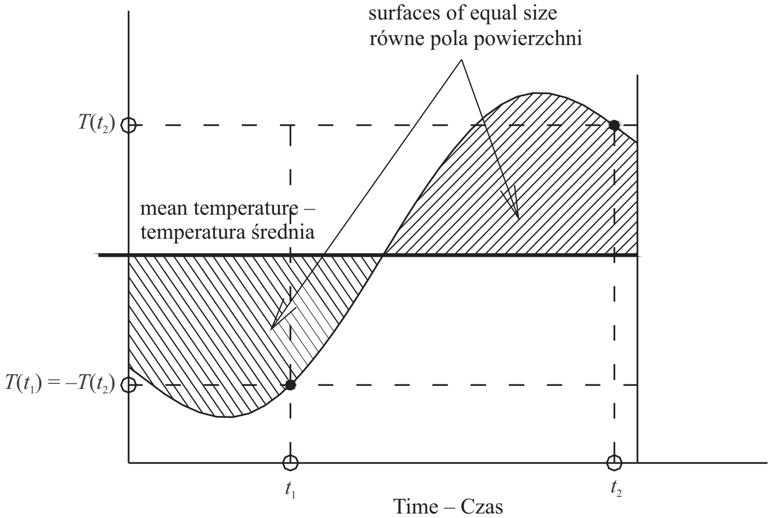


Fig. 3. Exemplary chart where daily mean temperatures, determined with formulas (1), (3) and (4) are identical.

Ryc. 3. Przykład sytuacji, gdy temperatury średnie dobowe, wyznaczone ze wzorów (1), (3) oraz (4) są jednakowe

## DAILY MEAN TEMPERATURE ON REAL-LIFE EXAMPLES

Actual temperature courses recorded by temperature measuring stations located in the eastern part of the Małopolska region, near the town of Chrzanów (N50.159539°, E19.384979°) will be used as practical examples, illustrating the scale of ambiguity in mean temperature calculations. The sampling rate was 1 minute. Given the fact that the highest significant values of temperature courses (from individual stations) are observed every few minutes, such a selection of the sampling rate allows for clear reconstruction of the temperature curve over time (in accordance with the Nyquist theorem [Bendat and Piersol 1971]). It can therefore be assumed that these courses, after suitable approximation in time, can be the basis for calculating daily mean temperature according to the formula (1), for an analog recorded function.

The temperature measurements were carried out with the help of electronic sensors located at different levels in relation to the ground surface. For the purposes of this study only some of the results have been used, from different periods of 2011, registered in the following measurement points:

- at a height of 1 m above the ground level,
- just above the ground level (at the height of approximately 5 cm) in the area artificially devoid of vegetation,
- just above the ground (like above) in the area covered with natural vegetation, without human intervention.

Daily mean temperature was determined on the basis of the obtained measurement data using three different definitions of the mean value: according to formula (1)–(3).

The most interesting examples of ambiguity in determining the mean value are shown in figures below.

Example presented in Fig. 4 illustrates the analyzed problem very well. In winter, the periods of intensive solar radiation are short, but the temperature can be increased thanks to the phenomenon of insolation (especially on surfaces that absorb solar radiation, such as the ground). With a certain time lag, the air is also heated. The graph of the actual temperature (Fig. 3) shows a sharp "peak" corresponding to such momentary, intense heating. Daily mean temperature, determined by the formula (3), equals 4.9°C in this case. If such information were to be reported by meteorological services, a false conclusion could arise that air temperature at the height of 1 m above the ground hovered around 4.9°C, because this is the way we intuitively perceive the concept of "mean temperature".

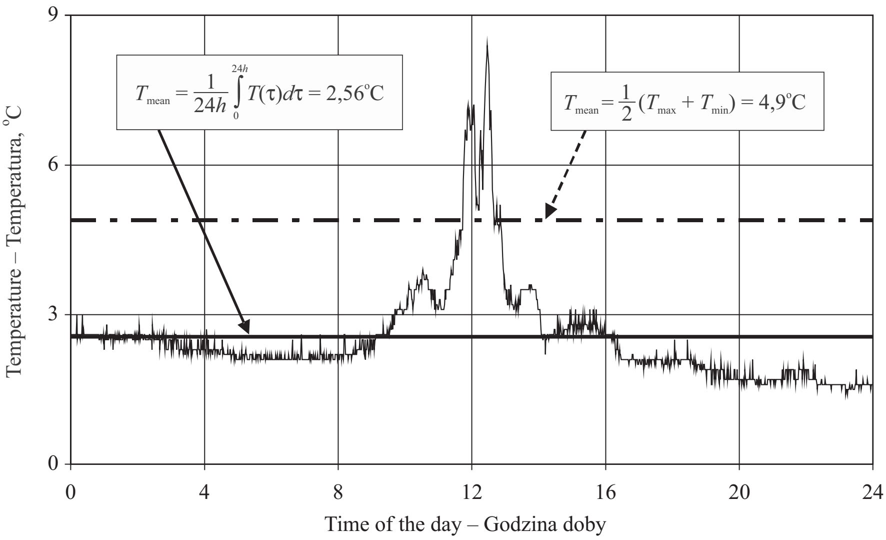


Fig. 4. Daily mean temperature on 20 January 2011, based on continuous course, measured at the height of 1 m above the ground

Ryc. 4. Temperatura średnia dobowa z 20 stycznia 2011 r. na podstawie przebiegu ciągłego; pomiar na wysokości 1 m nad powierzchnią ziemi

In fact (as shown in Fig. 4) the obtained mean values differ by 48%. The period of "heat", at the level of 4°C lasted for about 1.5 hours and for most of the day the temperature was beneath 3°C. In this case, it seems that the mean value provided by formula (1) describes thermal conditions on the analyzed day better. The conclusion that the temperature was around 2.6°C is acceptable.

It is worth noting that the mean value calculated according to formula (3) is 191.4% of the value calculated with formula (1). A similar challenge is encountered in the case of discrete measurements conducted every hour. This is presented in Fig. 5 based on the data from 20 January 2011 after data resampling (every 60th result was analyzed). The obtained average values differ by 47%.

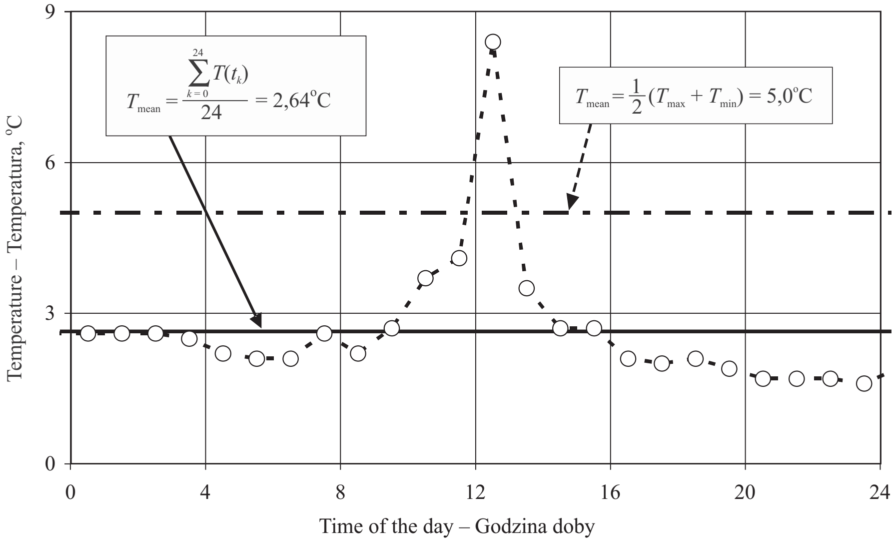


Fig. 5. Daily mean temperature on 20 January 2011 based on the results of hourly measurements at 1 m above the ground

Ryc. 5. Temperatura średnia dobowa z 20 stycznia 2011 r. na podstawie wyników pomiaru „co godzinę"; pomiar na wysokości 1 m nad powierzchnią ziemi

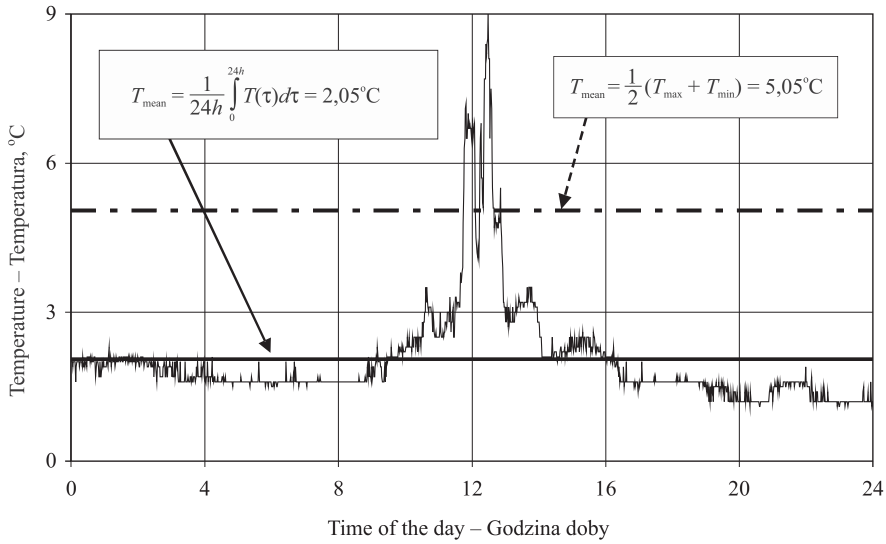


Fig. 6. Daily mean temperature on 20 January 2011, based on continuous progress, measured in an area artificially deprived of vegetation, 5 cm above the ground.

Ryc. 6. Temperatura średnia dobowa z 20 stycznia 2011 r. na podstawie przebiegu ciągłego; pomiar w terenie sztucznie pozbawionym roślinności, 5 cm nad powierzchnią ziemi

Differences in the calculated mean values are even more pronounced for the temperature measured on the ground surface and in the upper layer of soil, where the heat-induced transient effects of insolation are even stricter. This situation is illustrated in Fig. 6 and Fig. 7. In both cases, the mean value calculated according to formula (3) is more than 230% of the value according to formula (1).

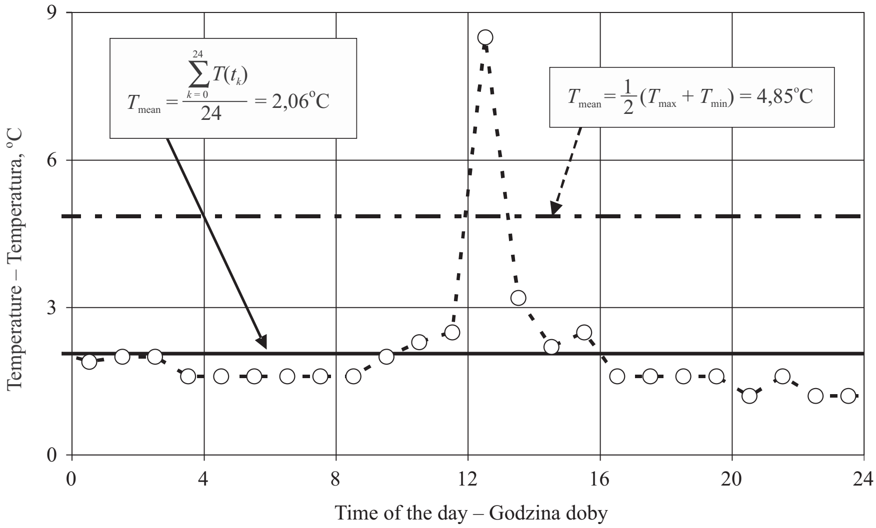


Fig. 7. Daily mean temperature values on 20 January 2011 based on hourly measurements in the area without vegetation, 5 cm above the ground. The obtained mean values differ by at least 58%

Ryc. 7. Wartości temperatury średniej dobowej z 20 stycznia 2011 r. na podstawie wyników pomiaru „co godzinę"; pomiar w terenie bez roślinności, 5 cm nad powierzchnią ziemi. Uzyskane wartości średnie różnią się o co najmniej 58%

The result of short-term, strong insolation can be also observed during the summer, in the periods of variable cloudiness. This is exemplified in Fig. 8 and Fig. 9.

Continuous temperature charts clearly illustrate periods (mainly in Fig. 7) when within one hour the temperature rapidly increased by more than 7°C, alternately decreased and increased by a few degrees in a few-minute cycles and then decreased to the initial value. Fig. 8 presents two sharp temperature drops by more than 7°C within a few minutes. In both cases, mean values calculated according to various formulas, differ by 3.5°K.

The examples above include graphs of temperature changes during one day. They make it possible to trace the effects of ambiguities in the calculation of mean temperature over a longer period. Fig. 10, 11 and 12 present monthly changes of daily mean temperature determined with formulas (1) and (3) for the month of April 2011, based on the data from several measurement points.

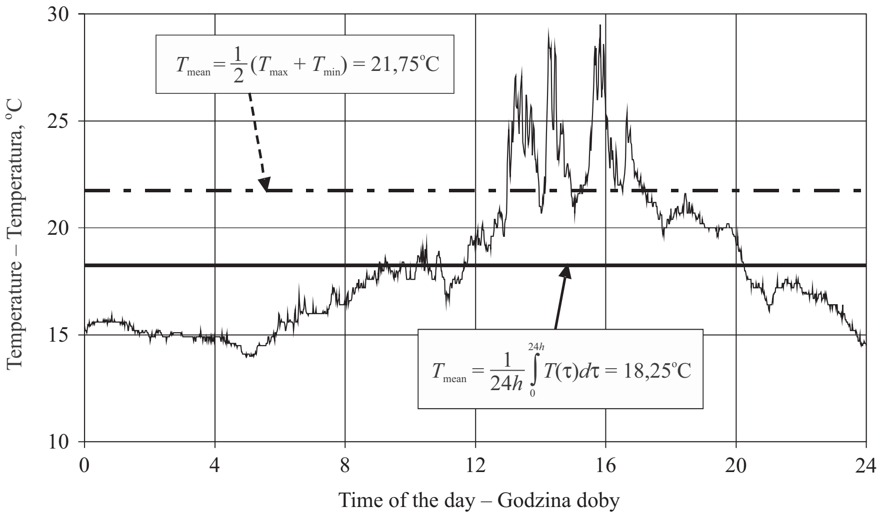


Fig. 8. Daily mean temperature on 5 July 2011, based on continuous progress, measured in the area without vegetation, 5 cm above the ground. The obtained mean values differ by more than 16%

Ryc. 8. Temperatura średnia dobowa z 5 lipca 2011 r. na podstawie przebiegu ciągłego; pomiar w terenie bez roślinności, 5 cm nad powierzchnią ziemi. Uzyskane wartości średnie różnią się o ponad 16%

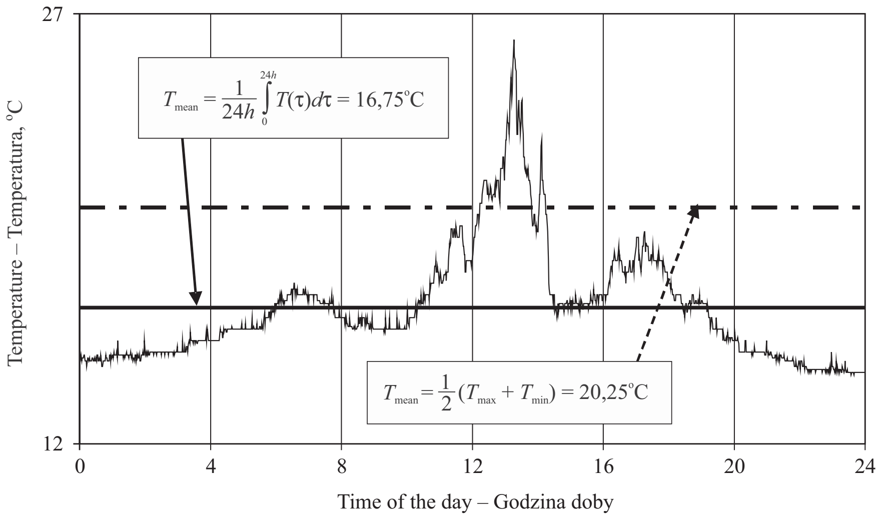


Fig. 9. Daily mean temperature on 14 June 2012 based on continuous progress, measured in the area devoid of vegetation, 5 cm above the ground. The obtained values differ by more than 17%

Ryc. 9 Temperatura średnia dobowa z 16 czerwca 2012 r. na podstawie przebiegu ciągłego; pomiar w terenie bez roślinności, 5 cm nad powierzchnią ziemi. Uzyskane wartości różnią się o ponad 17%

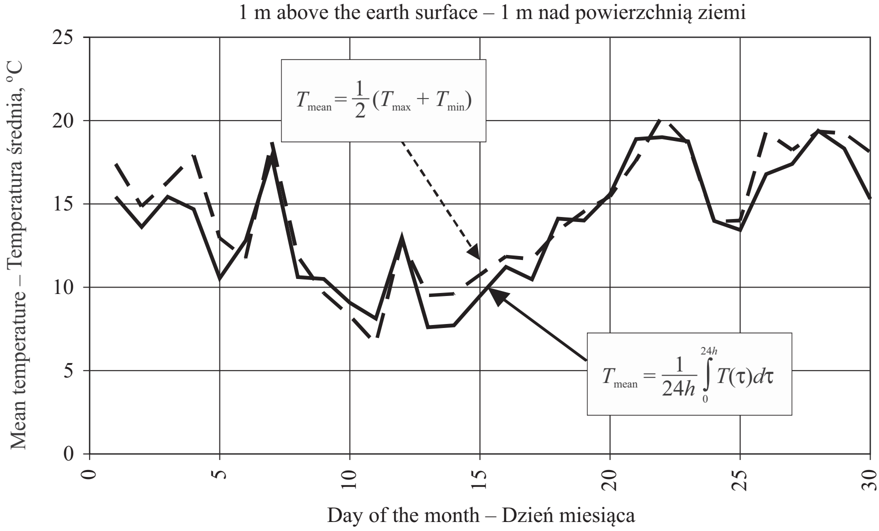


Fig. 10. Daily mean temperatures for April 2011, based on measurements conducted 1 m above the ground

Ryc. 10. Wykres temperatur średnich dobowych dla kwietnia 2011 r. na podstawie pomiaru na wysokości 1 m nad powierzchnią ziemi

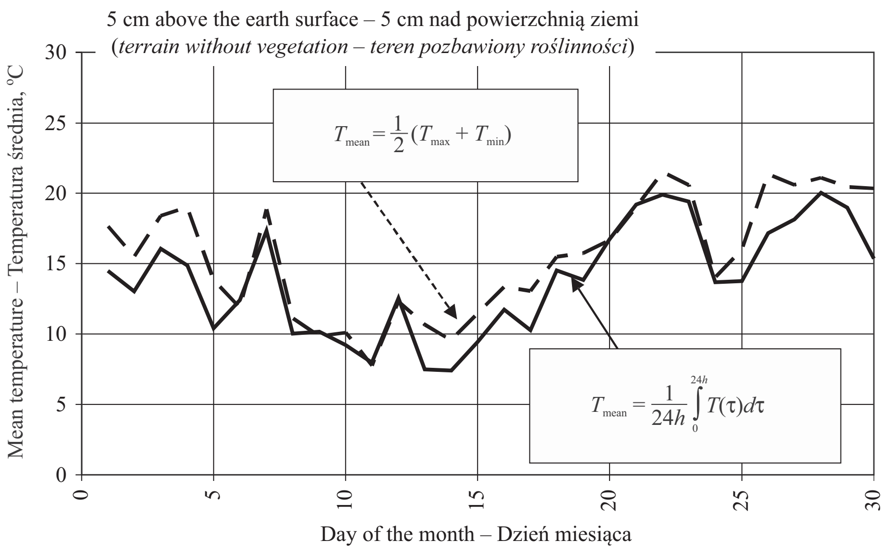


Fig. 11. Daily mean temperatures for April 2011, based on measurements conducted 5 cm above the ground in the area without vegetation

Ryc. 11. Wykres temperatur średnich dobowych dla kwietnia 2011 r. na podstawie pomiaru na wysokości 5 cm nad powierzchnią ziemi w terenie sztucznie pozbawionym roślinności

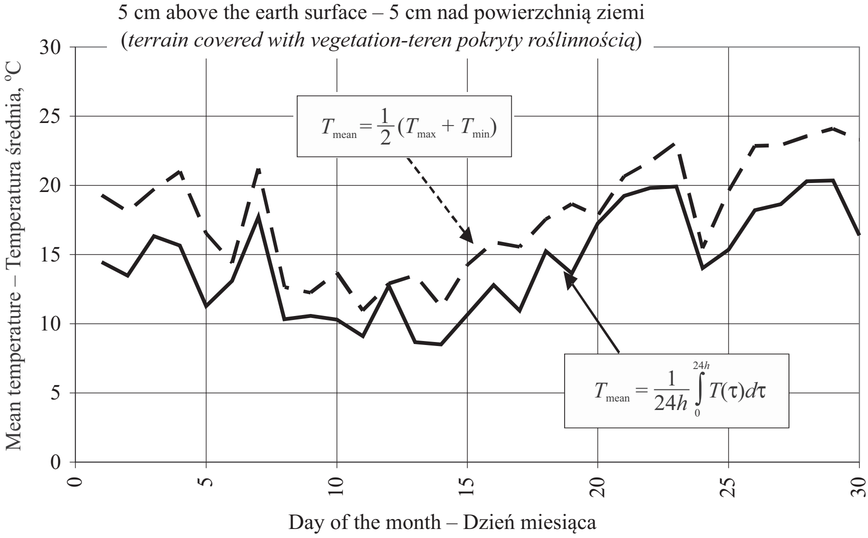


Fig. 12. Daily mean temperatures for April 2011, based on measurements conducted 5 cm above the ground in the area with vegetation (no human interference)

Ryc. 12. Wykres temperatur średnich dobowych dla kwietnia 2011 r. na podstawie pomiaru na wysokości 5 cm nad powierzchnią ziemi w terenie pokrytym naturalną roślinnością (bez ingerencji człowieka w teren)

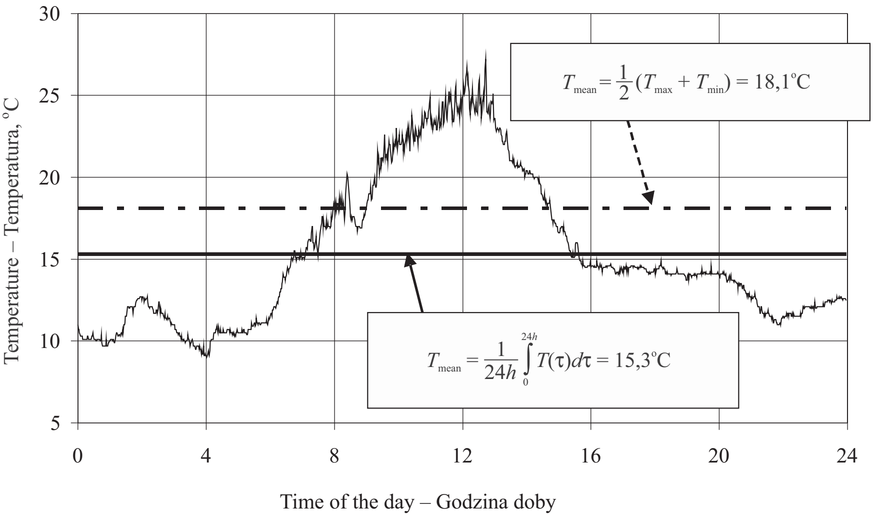


Fig. 13. Daily mean temperature values on 30 April 2011, based on continuous progress, measured at the height of 1 m above the ground

Ryc.13 Wartości temperatury średniej dobowej z 30 kwietnia 2011 r. na podstawie przebiegu ciągłego; pomiar na wysokości 1 m nad powierzchnią ziemi

Particularly significant differences in mean values can be observed on 30 April 2012. Subsequent figures present detailed temperature courses on that day with daily mean values designated by formulas (1) and (3).

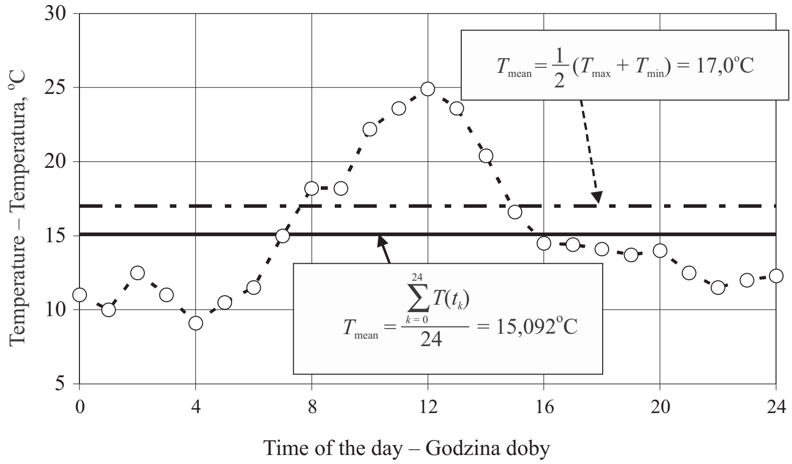


Fig. 14. Daily mean temperature values on 30 April 2011, based on hourly measurements at the height of 1 m above the ground

Ryc. 14. Wartości temperatury średniej dobowej z 30 kwietnia 2011 r. na podstawie wyników pomiaru „co godzinę"; pomiar na wysokości 1 m nad powierzchnią ziemi

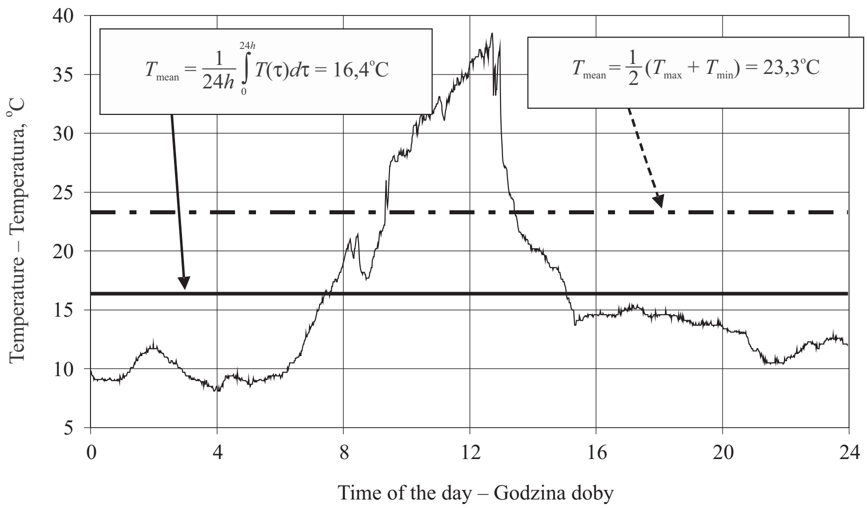


Fig. 15. Daily mean temperature on 30 April 2011, based on continuous progress, measured 5 cm above the ground in the area covered with vegetation. The difference in the obtained values is 7.9°C which is 29.6% of the higher value

Ryc. 15. Temperatura średnia dobowa z 30 kwietnia 2011 r. na podstawie przebiegu ciągłego; pomiar w terenie pokrytym roślinnością, 5 cm nad powierzchnią ziemi. Różnica w uzyskanych wartościach wynosi 7,9°K, co stanowi 29,6% większej spośród nich

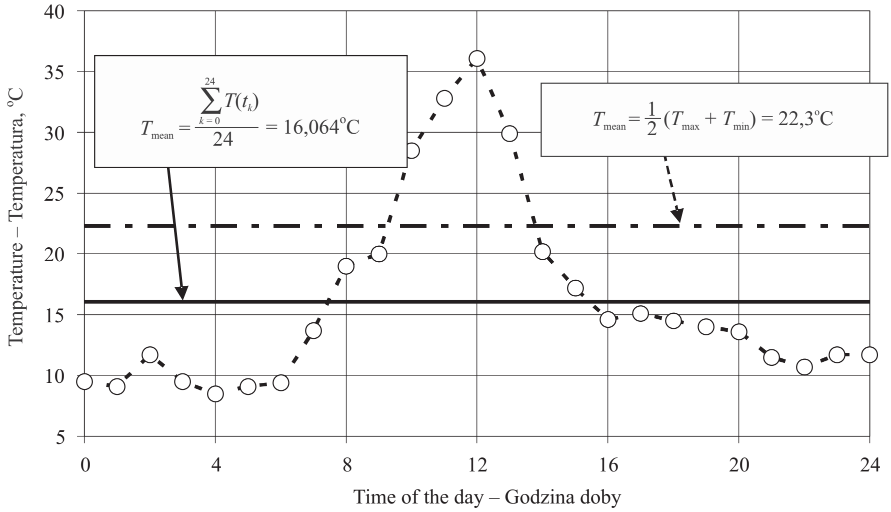


Fig. 16. Daily mean temperature values on 30 April 2011, based on hourly measurements in the area covered with vegetation, 5 cm above the ground

Ryc. 16. Wartości temperatury średniej dobowej z 30 kwietnia 2011 r. na podstawie wyników pomiaru „co godzinę"; pomiar w terenie pokrytym roślinnością, 5 cm nad powierzchnią ziemi

## CONCLUSIONS

The concept of daily mean temperature is highly ambiguous. Various mathematical formulas used for calculating mean values, lead to results that differ by many Kelvin degrees, and the relative error, referred to the greater of the obtained values can reach many percent. Particularly large discrepancies appear when:

- the analysis concerns a highly overcast day, with short clearings,
- the analysis concerns sunny days in winter, when the period of intense insolation is short,
- the calculated mean temperature refers to the ground surface, air layers just above the ground level or other places located near objects absorbing solar radiation (e.g., near the walls of buildings),
- there are other factors causing short-lived but significant changes, of a momentary character (e.g., hail, which temporarily but significantly lowers the temperature of the ground surface).

The mentioned discrepancy in the obtained mean values, with an error at the level of several percent, puts into question the possibility of comparing results without a clear and precise definition of the conditions under which the study was conducted. We must therefore, in any case, to answer the following questions:

- What was the method of temperature recording (analog or digital)?
- Was the measurement digital – what was the sampling period?
- Does the applied sampling frequency meet the Nyquist demand?

- What is the definition of mean value (what mathematical formula is used)?
- Have there been any attempts at determining and comparing the mean according to different definitions? In certain situations, as previously mentioned, different definitions of mean may lead to comparable or identical values. For example, this may be the case for measurement and analysis of soil conditions in deeper layers, where the temperature curve is almost harmonic (Fig. 2).

As long as there are no answers to the questions above, it is difficult or even impossible to carry out scientific comparisons in climatologic and agronomic studies. Researches should be particularly wary of using historical climate data, if the conditions under which the data originated are unknown.

## REFERENCES

Allen J.C., 1988. Averaging functions in a variable environment: a second order approximation method. Environ. Entomol. 17, 621–625.

Bendat J.S., Piersol A.G., 1971. Analysis and measurement procedures. Wiley-Interscience, John Wiley&Sons, Inc. New York – London – Sydney – Toronto.

Котельников В.А., 1933. О пропускной способности “эфира” и проволоки в электросвязи. Материалы к всесоюзному съезду по вопросам реконструкции дела связи и развития слаботочной промышленности. Всесоюзный Энергетический Комитет Москва.

Nyquist H., 1928. Certain topics in telegraph transmission theory. Trans. AIEE.

Shannon C.E., 1949. Communication in the presence of noise. Proc. of IRA, No. 1.

Tijskens L.M.M., Verdenius F., 2000. Summing up dynamics: modelling biological processes in variable temperature scenarios. Agric. Systems 66, ELSEVIER, 1–15.