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PHYSICAL AND MATHEMATICAL PONDERABILITY OF TEMPERATURE MEASUREMENT'S FREQUENCY IN ENVIRONMENTAL ENGINEERING

FIZYCZNA I MATEMATYCZNA WAŻKOŚĆ CZĘSTOTLIWOŚCI POMIARU TEMPERATURY W INŻYNIERII ŚRODOWISKA

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Abstract. The paper discusses circumstances that lead to improper measurements of temperature: without appropriate consideration to the context of physical phenomena occurring in nature or without appropriate awareness of mathematical principles; all of which results in data misinterpretation. Based on the conducted discussion, it can be stated that measurements and analyses of thermal phenomena, which are time-varying processes, should be performed using the same principles and tools as in the identification of dynamic processes, such as the Nyquist theorem.

Streszczenie. W pracy rozważane są okoliczności, w których dyskretne pomiary temperatury prowadzone są przy niewłaściwie dobranym kroku dyskretyzacji, bez należytego uwzględnienia kontekstu fizycznego zjawisk przebiegających w przyrodzie oraz bez świadomości uwarunkowań matematycznych, co prowadzi do błędnej interpretacji uzyskanych danych. Z przeprowadzonych rozważań wyprowadza się postulat, aby pomiary i analizy zjawisk termicznych, jako procesów zmiennych w czasie, były prowadzone z wykorzystaniem zasad i narzędzi stosowanych w identyfikacji procesów dynamicznych, m.in. twierdzenia Nyquista.

Key words: Nyquist theorem, temperature measurement, soil temperature

Słowa kluczowe: twierdzenie Nyquista, pomiar temperatury, temperatura gleby

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INTRODUCTION

Recent years have faced a growing interest in the modeling of biological processes which occur in dynamically changing environments. For example, scientists have been developing models to identify the rate of weight gain (e.g. in white beet, potato or corn grain) depending on humidity changes, the availability of macro- and micronutrients as well as temperature changes. As a result, they have developed mathematical procedures describing relevant operational plant (in other cases also animal) parameters, as functions of environmental variables. Such procedures can be dynamic (time-history analysis), then time becomes one of the variables; or they can be simple mathematical formulas of empirical character with e.g. average temperature (of air, soil at different depths, etc.) as their variable [Tijskens and Verdenius 2000]. In order to measure temperature as a variable in a dynamic process it is necessary to be acquainted with the basic principles for the selection of measurement frequency, such as the Nyquist theorem [Nyquist 1928]. The procedure of averaging temperature as a variable in empirical formulas requires the use of standardized methods. It is at least desirable to realize the margin of error when averaging the data obtained by different methods.

DISCRETE MEASUREMENT OF A CONTINUOUS PROCESS

Let us assume that a scientist performs measurements of a physical quantity changing in time in a continuous manner by recording its value at certain intervals. Dynamic analysis uses the term "sampling period" or "sampling rate", i.e. the reciprocal of the period (number of readings per unit of time). Let this be temperature measurement. The results (theoretically, without providing any value) are represented in Fig.1. If the measured temperature is a variable of a certain bio-process, we need to know its exact course throughout the entire measurement period. However, do the measurement data make it possible to reconstruct unequivocally temperature values at the time the measurement was carried out?

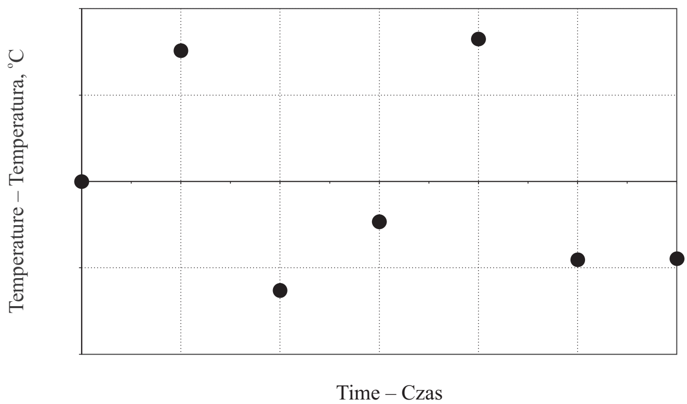

Fig. 1. Hypothetical results of a discrete temperature measurement in a certain time-period
Ryc. 1. Hipotetyczny wynik dyskretnego pomiaru temperatury w pewnym okresie czasu

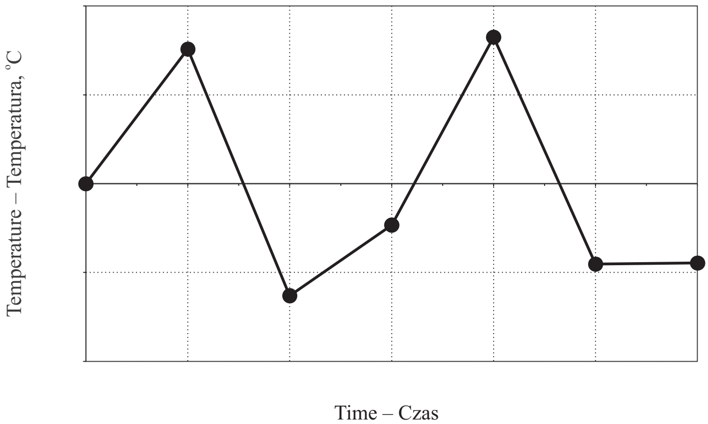

Fig. 2. Simple linear approximation of measurement results
Ryc. 2. Prosta aproksymacja liniowa dyskretnych wyników pomiaru

Contrary to what may appear, this is not a minor question. Very often in such cases, the researcher uses linear approximation, assuming that the temperature had "average" values in the periods of time between consecutive measurements (Fig. 2).

If the aim is to determine a smooth curve (the process is continuous), a spline function approximation is usually made with the help of the Chebyshev or Hermite polynomials or linear combinations of trigonometric functions. Examples of such approximations can be seen in Fig. 3.

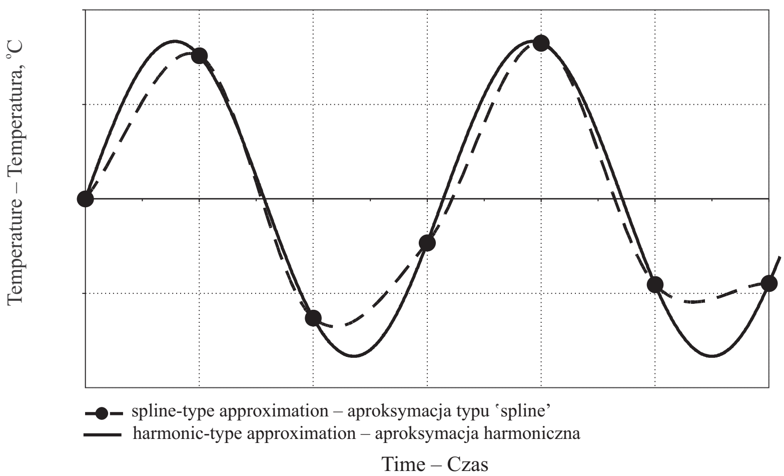

Fig. 3. Simple approximation of discrete measurement results assuming that the process does not include elements of high frequency
Ryc. 3. Przykłady prostej aproksymacji dyskretnego wyniku pomiarów przy założeniu braku składowych procesu o wysokich częstotliwościach

The choice of trigonometric harmonic functions for the approximation of temperature results seems to be justifiable and quite common. We conduct a discrete measurement and assume that its results reflect a certain process which includes periodic parameters; that there is a "hidden" repeatability in the process. The term "harmonic" is sometimes used in this context, to refer to the *harmonic element of the process.* Such reasoning is physically justified and confirmed empirically. Observing the temperature change over time in real life conditions, we can notice a cyclical nature (periodicity) of these changes. This is a result of the apparent movement of the sun across the horizon (cyclical daily), or the angle of the sun to the earth's surface (annual periodicity). So, referring to the previously introduced concepts, it seems reasonable to make an attempt at approximating temperature with the help of at least two harmonics: daily and annual.

Is that enough though? In cloudy weather, for example, temperature variations, especially near the earth surface and in top layers of soil, are strongly influenced by radiation variability. Changes in temperature can also be associated with other physical phenomena, such as convection, radiation, rainfall, or water evaporation. All of these processes are dynamic and change over time. Their superposition can lead to the illusion of cyclicality. It can be therefore expected that there will be other harmonics in temperature course, which would be very difficult to identify unequivocally. Figure 4 illustrates examples of possible results of harmonic approximation of the measurement results presented in Fig. 1. Several functions, of different periods, have been applied here. In fact, there is an infinite number of these functions. We can therefore refer here to basic harmonics resulting from the simplest harmonic approximation (Fig. 3) and an infinite number of further components, with shorter and shorter periods.

If the measurement points to the fundamental harmonic frequency f_0, it can be assumed that any process of harmonic frequency presented below can be a real process:

$$f = k \cdot f_S \pm f_0 \tag{1}$$

where: k – natural number,
f_S – sampling frequency during the measurement.

For example: if the measured discrete values suggest that temperature changes were cyclical, that is $f_0 = (24)^{-1} = 0.04166\ \mathrm{h}^{-1}$ and temperature readings we conducted every hour, that is $f_S = 1{,}0\ \mathrm{h}^{-1}$ it cannot be excluded that the cyclicity of real temperature changes can be: 1.04166; 2.04166; 3.04166 h^{-1}, etc. We can call them "hidden" frequencies, because measurement readings do not make it possible to identify the frequency unambiguously. This is due to a well-known phenomenon called "aliasing", often encountered when it is difficult to identify dynamic objects (*alias* = Lat. *otherwise*).

Based on the data obtained at large time intervals, not only is it impossible to reconstruct the real-time process, but in some cases, the data can lead us to an erroneous interpretation of the phenomenon. This can be exemplified by data presented in Fig. 5, where a quick changeable process (let us assume that it is temperature) was recorded by two researchers with the same, yet very low frequency. However, one of the researchers started the measurements slightly earlier. The result of the first measurement (1) indicates that initially the temperature decreased, after reaching a certain value it started to increase and then fell again. The other measurement results (2) suggest quite the opposite. This is very confusing since both researchers used an efficient measuring apparatus and registered the results in the same period of time.

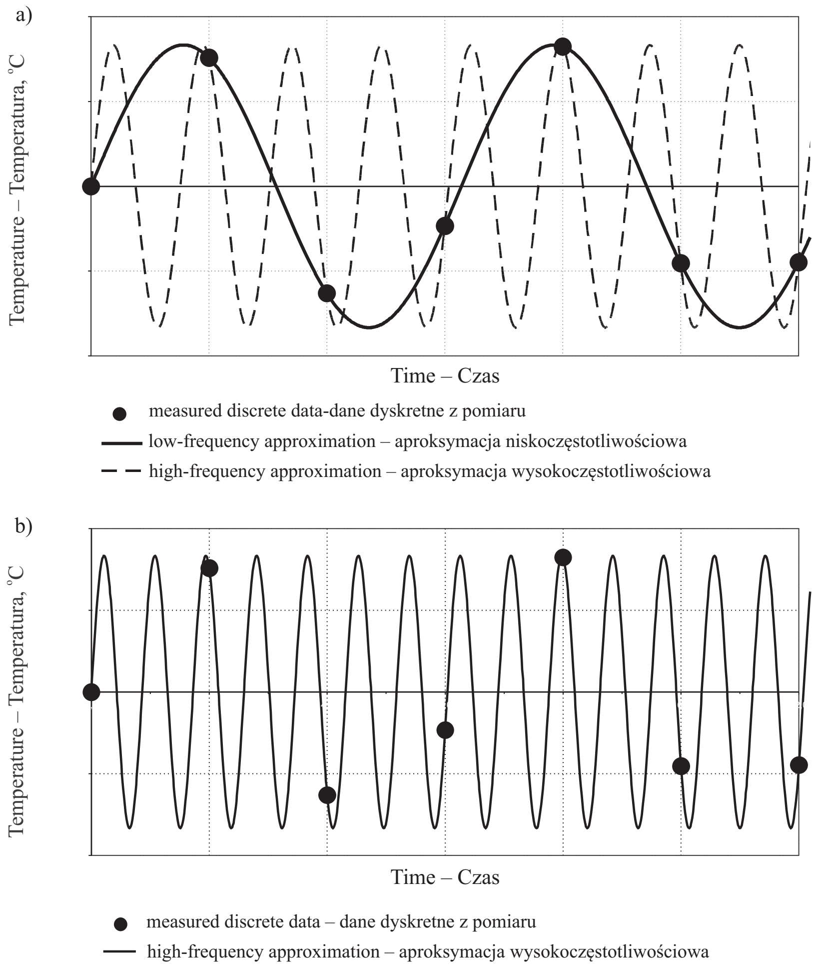

Fig. 4. Exemplary possible variants of discrete measurement approximation with the help of harmonics with different frequencies

Ryc. 4. Przykładowe, możliwe warianty aproksymacji pomiarów dyskretnych za pomocą harmonik o różnych częstotliwościach

In turn, Fig. 6 presents results of two variants of discrete measurements, conducted in two different, very low sampling rates. The results marked as (1) suggest that during the entire study period the temperature assumed only positive values. The measurement results marked as (2) points to a cyclic temperature change with low-frequency, which has nothing to do with the actual course of events.

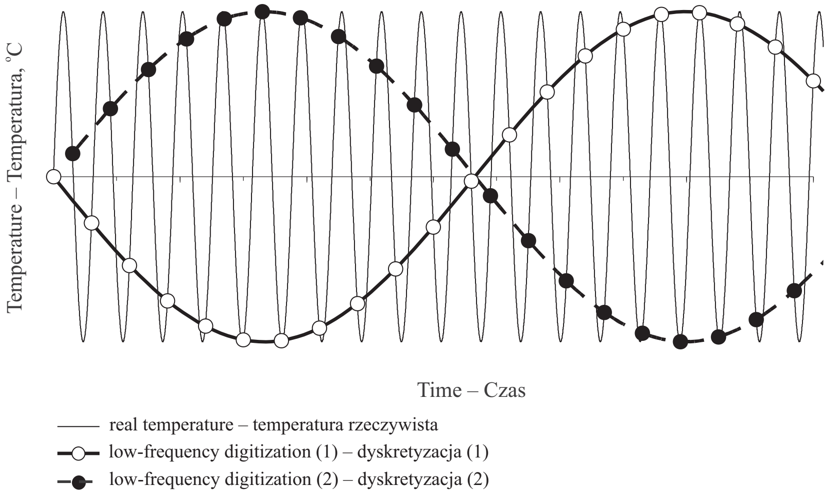

Fig. 5. Example of possible erroneous interpretations of monotonicity phenomena based on discrete measurements

Ryc. 5. Przykładowe, możliwe błędne interpretacje monotoniczności zjawiska na podstawie pomiarów dyskretnych

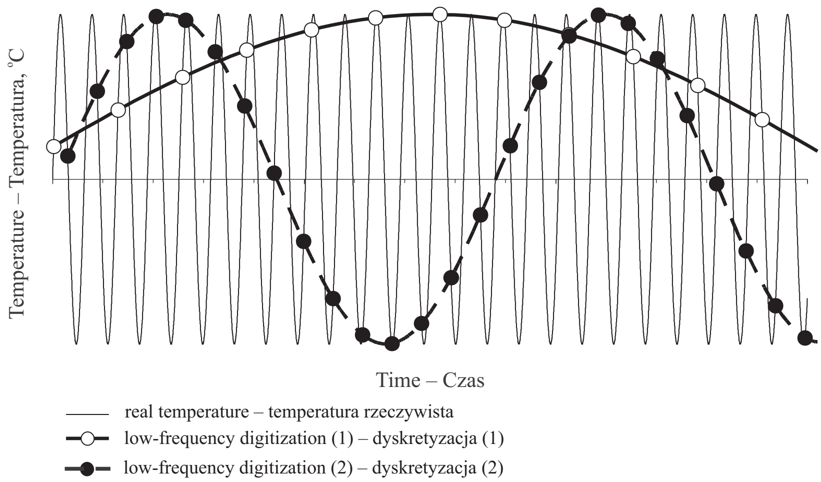

Fig. 6. Examples of possible erroneous interpretations of the phenomenon character based on discrete measurements

Ryc. 6. Przykładowe, możliwe błędne interpretacje znaku zjawiska na podstawie pomiarów dyskretnych

Fig. 7 presents possible misinterpretations of amplitude as a result of too low sampling frequency. In both cases, the frequency of the actual process was interpreted incorrectly as well (the previously mentioned phenomenon of “aliasing”).

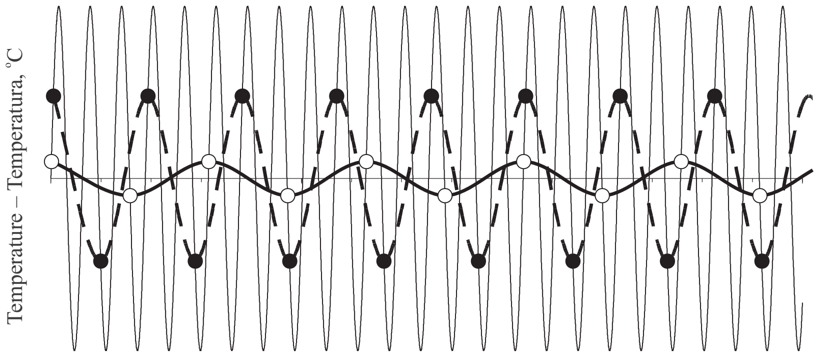

Fig. 7. Examples of possible erroneous interpretations of frequency and amplitude
Ryc. 7. Przykładowe, możliwe błędne interpretacje częstotliwości i amplitudy zjawiska

Let us imagine a situation in which two researchers are trying to register the course of a harmonic phenomenon (e.g. temperature), with high frequency. Both researchers assume that the sampling period is equal to the process harmonics or its multiple, because they do not know the real nature of the phenomenon. Such a situation can happen by accident, it is unlikely and purely hypothetical, but possible. The results of such measurements and the consequent interpretations of reality are shown in Fig. 8.

Properly carried out measurements of a continuous dynamic process, including temperature, should comply with a valid demand (towards the sampling frequency), first formulated by Nyquist [1928], and later also by Kotielnikov [1933] and Shannon [1949]:

> a continuous process can be correctly reconstructed through an analysis of discrete measurement data, including all components of the variables over time, if the sampling rate f_S during the measurement exceeds at least twice the highest harmonics contained in the process, namely:

$$f_S \geq 2 \cdot f_{\max} \tag{2}$$

This requirement is known as the Nyquist theorem, from the name of its first author. The calculated and required sampling frequency is called the Nyquist frequency [Bendat and Piersol 1971].

Previous considerations over the wrong interpretation of temperature measurements may seem purely theoretical; it may be questionable to assume the existence of components of high-frequency in temperature courses. It must be mentioned that temperature measurements are often carried out at intervals of several hours and sometimes twice a day; in such situations, the process component with a few or several-second period can be referred to with the term "high frequency". It is also worth tracing the results of disregarding the Nyquist theorem in temperature measurements in practical examples.

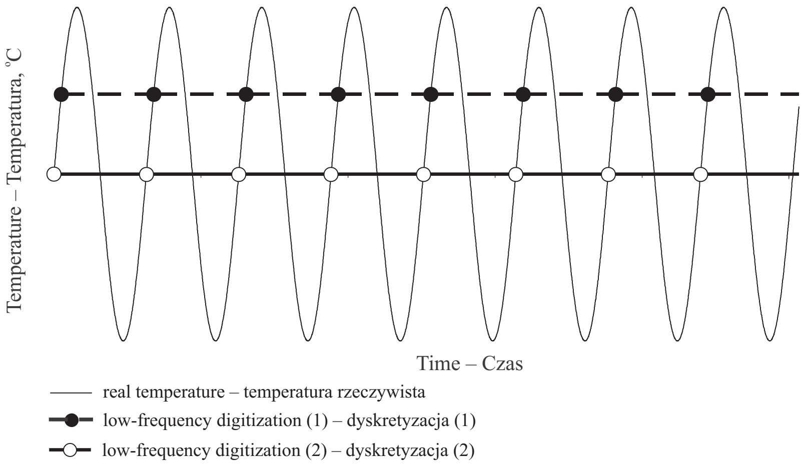

Fig. 8. The effects of random compliance of sampling frequency with the process harmonics
Ryc. 8. Skutki przypadkowej zgodności częstotliwości próbkowania z częstotliwością procesu harmonicznego

IMPORTANCE OF THE NUQUIST THEOREM IN REAL LIFE MEASUREMENTS

In order to realize how important it is to select the right sampling frequency (number of measurements per unit of time) let us analyze real-life temperature courses registered at measurement stations located around the town of Chrzanów in the western part of the Malopolska region (N 50.159539°, E 19.384979°). The sampling frequency was 1 minute. It was adopted on the basis of previous studies, which showed that the highest, significant components of the temperature course have a few-minute rates. The Nyquist demand was thus fulfilled. For the purposes of this presentation it is therefore assumed that these courses will be referred to as "precise" and "real", as if they corresponded to analog signals. Measurements were carried out with the help of electronic sensors located at different levels in relation to the ground surface. This study uses the data recorded just above the ground, at a height of about 5 cm, at different periods of December 2010.

On the basis of the data presented above, an attempt has been made to reconstruct the results of measurements on the assumption that the sampling time equaled 1 hour. Thus, every sixtieth registered value was assumed as the presumed measurement result. Of course, such a reconstruction attempt can have an infinite number of variations, with different starting points; the presented result should therefore be considered as an example.

Fig. 9 illustrates very well what happens when we describe a continuous phenomenon with discrete data, recorded at one-hour intervals. The points on the time axis correspond to the measurement time. A few-minute frequencies, which can be seen in the real-life course of the phenomenon, are completely "lost" due to excessively long sampling periods. The differences between the actual values and the presumed temperature, approximated on the basis of discrete data, are > 4°K in certain time-periods.

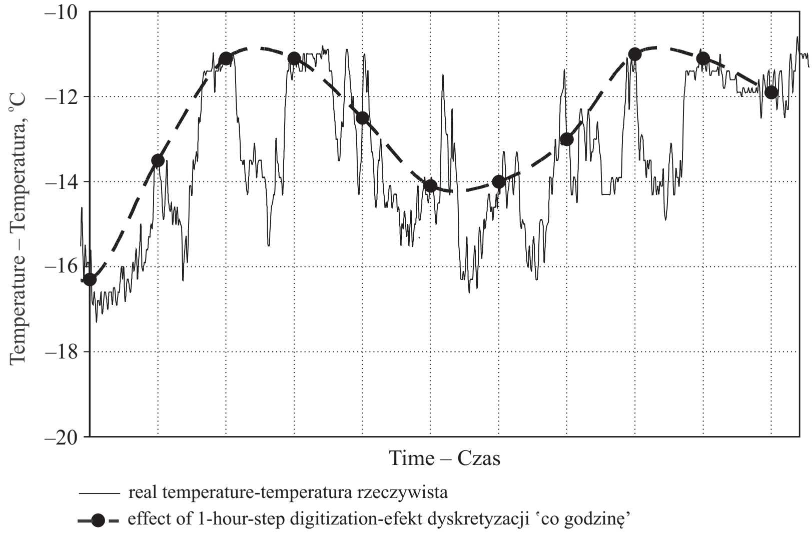

Fig. 9. Reconstruction of temperature measurement results (5 cm above the ground) against the actual course (16 December 2010, Poland N50.159539°, E19.384979°)

Ryc. 9. Rekonstrukcja wyników pomiaru temperatury (5 cm nad powierzchnią ziemi) na tle jej rzeczywistego przebiegu (16 grudnia 2010, Polska N50,159539°, E19,384979°)

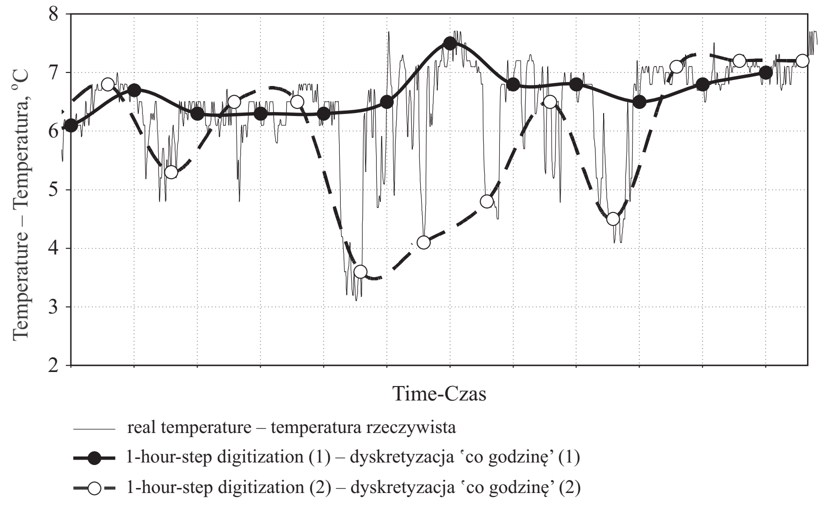

Fig. 10. Reconstruction of temperature measurement results against the actual course (24 December 2010, Poland N50.159539°, E19.384979°)

Ryc. 10. Rekonstrukcja wyników pomiaru temperatury na tle jej rzeczywistego przebiegu (24 grudnia 2010, Polska N50,159539°, E19,384979°)

Fig. 10 shows reconstructions of temperature course on the basis of measurement data from two recording devices. Both sets of discrete data come from the same point, correspond to the same period of time and were recorded at 1-hour intervals. However, the measurements in point 1 started 35 minutes earlier than in point 2. Reconstructed temperature courses corresponding to both those groups do not coincide; in certain periods of time, the differences between the two graphs amount to 3.5°K.

Fig. 11 shows two different reconstructions of temperature on the basis of two different measurements carried out in the same place and at the same time; in both cases a 1-hour sampling period was applied and the only difference is a time shift between the moment the measurements started. Interpretations of the two graphs are, in a way, mutually contradictory. Periods of temperature growth in graph (1) correspond to the periods of the temperature drop in graph (2) and vice versa. If such data were to be used as a variable in the description of a dynamic phenomena, we will have a situation in which researcher 1 will relate the course of the phenomenon with increasing temperature, while the researcher 2 will relate the course of the studied phenomenon with temperature decrease, because, according to this measurement data, the decrease in time has occurred. As a result, there will be two mutually contradictory theories despite the fact that the average values of the two measurements are comparable.

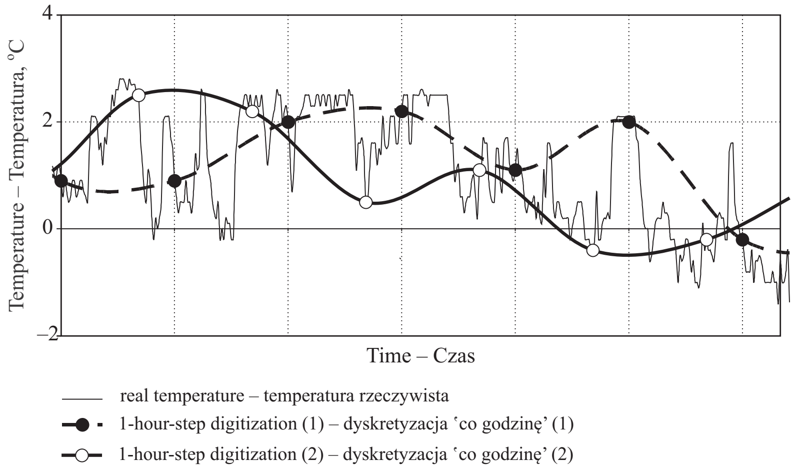

Fig. 11. Reconstruction of temperature measurement results (5 cm above the ground) against the actual course (22 December 2010, Poland N 50.159539°, E 19.384979°).

Ryc. 11. Rekonstrukcja wyników pomiaru temperatury (5 cm nad powierzchnią ziemi) na tle jej rzeczywistego przebiegu (22 grudnia 2010, Polska N50,159539°, E19,384979°)

It may be also the case that the results of two different measurements in the same period of time, lead to two different average values that differ significantly. An exemplary situation is presented in Fig. 12, where in a given period, the average calculated as a result of measurement marked as (1) is –14.5°C, while for measurement (2) it equals –12.5°C. So the four-hour measurement, conducted by two researchers, leads to two different average values differing by 2°K, which is about 14% of the higher value.

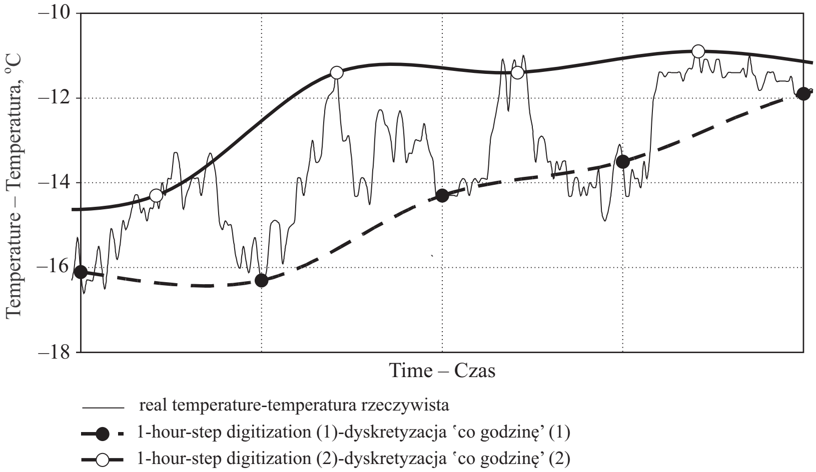

Fig. 12. Reconstruction of temperature measurement results (5 cm above the ground) against the actual course (17 December 2010, Poland N50.159539˚, E19.384979˚)

Ryc. 12. Rekonstrukcja wyników pomiaru temperatury (5 cm nad powierzchnią ziemi) na tle jej rzeczywistego przebiegu (17 grudnia 2010; Polska N50,159539˚, E19,384979˚)

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